NEW
FAVOURITE
MATH

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New Favourite Math carefully observes:

- using mathematical concepts, generalizations and laws in a smooth way.
- employing points of stimulating all forms of mathematical thinking.
- age, in addition to the developmental and physiological features of the students.
- employing modern constructivist teaching methods.
- using realistic pictures and portrayals far away from fantasy.
- connecting mathematics with life to highlight the importance of mathematics through mathematical problems.
- designing teaching resources and aids.
- the modernity of the content and keeping up with the technological advancement.
- creating the spirit of challenge and competition.
- steering clear of routine and unnecessary repetition.

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Level F

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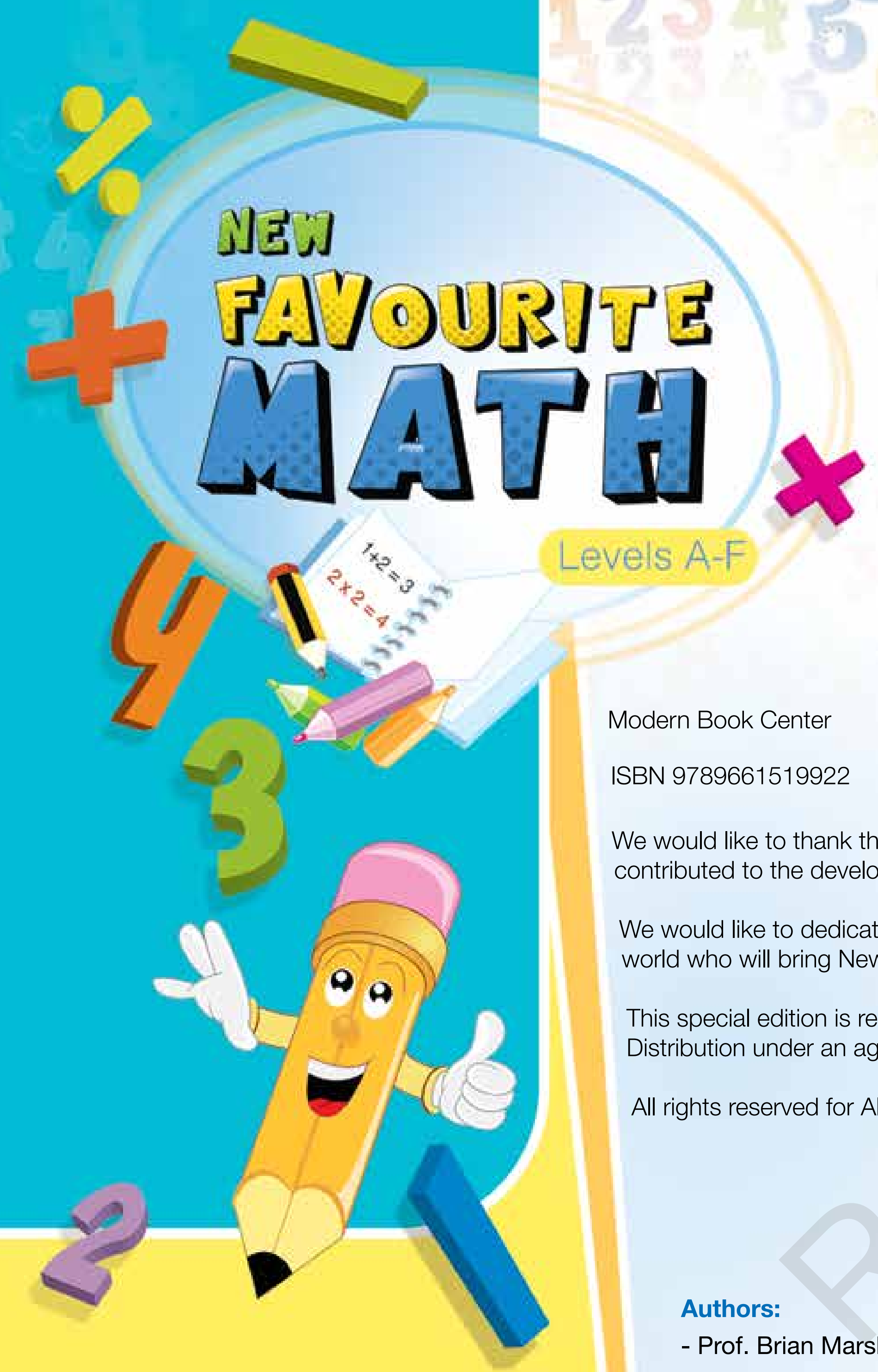

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We would like to thank the editors and our designers, who all contributed to the development of New Favourite Math.

We would like to dedicate this course to the teachers around the world who will bring New Favourite Math to life in their classrooms.

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NEW FAVOURITE MATH

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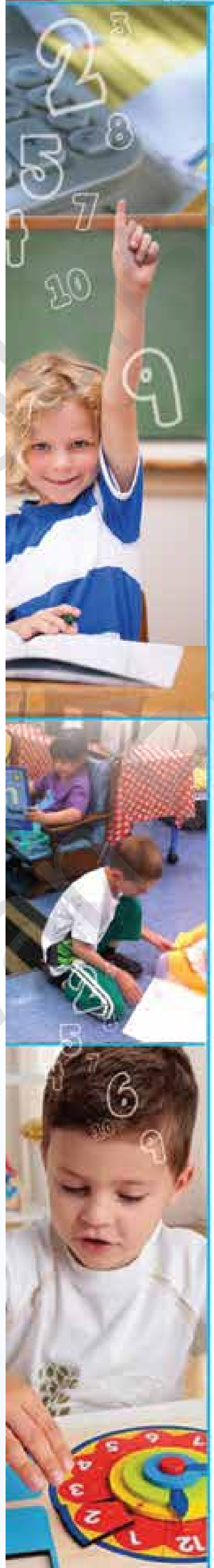


New Math 6th Grade

Problem Solving

The strategies used in solving word problems:

1. What do you know?

2. What do you need to know?

3. Draw a diagram/picture.

- **Solve the following word problems using the guess-and-check problem solving strategy to help solve math word problems.**
- **Make a table and look for a pattern.**

1. Identify - What is the question?
2. Plan - What strategy will I use to solve the problem?
3. Solve - Carry out your plan.
4. Verify - Does my answer make sense?

- **Find a pattern model.**

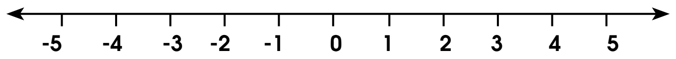
  Find a pattern method of problem solving strategy.

- **Solving word problems:**

Step 1: Identify (What is being asked?)
Step 2: Write the equation(s)
Step 3: Answer the question
Step 4: Check

1.1 Integers

Look at the Line Number.
Do you see any fractions?
Do you see any mixed numbers?



A number with no fractional part is called an integer.

So, integers can be:
- Counting numbers {1, 2, 3, ...}
- Zero {0}
- Negative numbers {-1, -2, -3, ...}

We can write them all down like this: {..., -3, -2, -1, 0, 1, 2, 3, ...}
Examples of integers: -16, -3, 0, 1, 198

- **Natural numbers 1, 2, 3, 4, …….. are also called positive integers.**

The Concept of Smaller and Greater Integers

The symbol (**<**) means ‘less than,’ and the symbol (**>**) means ‘greater than’. The integers shown on the line number are written in the order of magnitude, so that the integers increase to the right. For example, 3 is to the right from 1, so it is greater than 1, and written as 3 > 1. Similarly, 0 is to the right of -2, so it is greater than -2, and written as 0 > -2. The same property of order in integers extends right across the number line of integers. The order property can be stated in a reverse manner, so that the integers decrease to the left.

As we move to the left, integers get smaller. On the other hand, when we move to the right, integers get bigger.

Exercises

1 Fill in the blanks by using the symbols > (greater than) or < (less than).

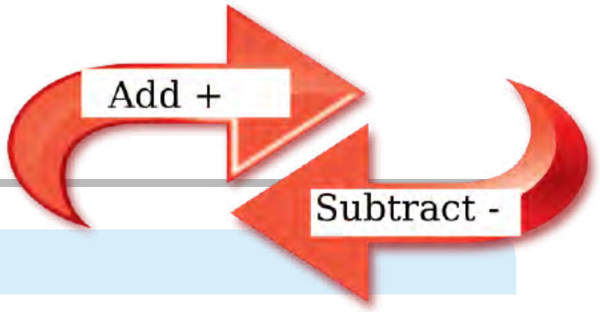
a -4 $\boxed{<}$ 0   b -7 $\boxed{<}$ 5   c -9 $\boxed{<}$ -7

2 Arrange the following numbers in ascending order.

6, 5, -6, 0, -2, 6, 13, -12, -13, -18   -18 , -13, -12, -6, -2, 0, 5, 6

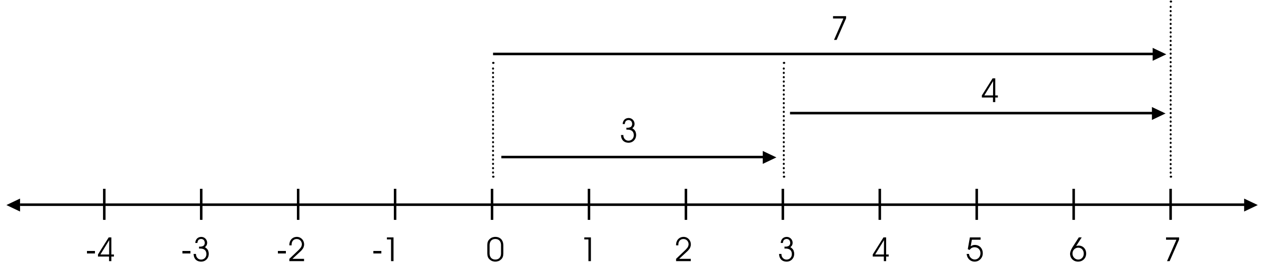
3 Arrange the following numbers in descending order.

-9, -12, -7, 7, -16, 11, 5, -8, 16, 9, -1   16, 11, 9, 7, 5, -1, -7, -8, -9, -12, -16



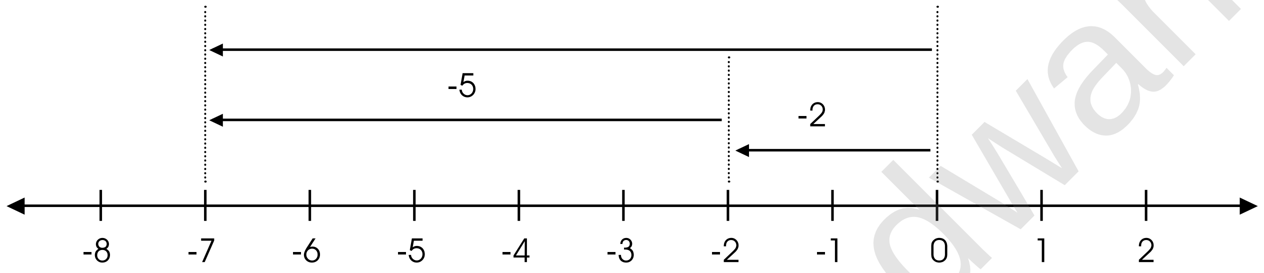
1.2 Addition of Integers

We use the number line to illustrate the addition of 3 and 4.
We start at 0 and proceed three units to the right to 3. We then proceed four more units to the right; this brings us to 7. So 3 + 4 = 7



Now, let us consider the addition of negative numbers, e.g. -2 and -5.
To add the negative integers -2 and -5 on the number line, we start at 0 and proceed two units to the left to -2. Then we proceed five more units to the left; this brings us to -7, (-2) + (-5) = -7

-7 = (-2)+ (-5)



Notice that the sum of two negative integers is the negative of the sum of their numerical values. i.e.
(-2) + (-5) = -2 -5 = -(2+5) = -7

Exercise

Find the sum of the following:

a -3 + (-8)= -11
b -75 + (-25)= -100
c -3 + (-8)= -11
d -75 + (-25)= -100

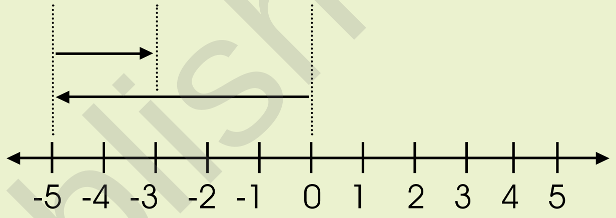
1.3 Subtraction of Integers

When subtraction involves a negative number, we should learn some rules.
Consider the following examples: Subtract -2 from 6 i.e. 6 - (-2) = 8.
When two negative signs (-) appear with the same number, it becomes positive. 6 - (-2) = 6 + 2 = 8.
So, when subtracting a negative number, the number becomes positive.

Find the result of the following operations:

e.g.

a (-5) - (-2) = -5 + 2 = -3



b (-8) - (-6) = -8 + 6 = -2
Show it on the line number

c 8 - (-6) = 8 + 6 = 14
Show it on the line number

Exercises

d (-200) - (-25) = -175
e 250 + 100 - (-100) = 450
f (-14) - (-254) = 240
g (-200) - (-25) = -175
h 23 - 500 +120= -357
i 61 - 100= -39

1.4 Multiplication of Integers

Rule 1:

When multiplying two integers that have the **SAME** sign, the answer is **POSITIVE**.

Positive x Positive = Positive

(+7) x (+10) = (+70)

Negative x Negative = Positive

(-5) x (–5) = (+25)

Rule 2:

When multiplying two integers that have **DIFFERENT** signs, the answer is **NEGATIVE**.

Positive x Negative = Negative

(+8) x (–6) = (-48)

Negative x Positive = Negative

(-5) x (+7) = (-35)

Exercise

Find the product of each of the following:

a -12 X 2 = -24

b 11 X – 11 X -1 = 121

c -20 X -10 = 200

d 7 X 8 X -10 = -560

e 8 X -10 X 2 = -160

f -3 X 2 X -1 X 10 = 60

1.5 Division of Integers

If both the dividend and divisor are **positive** or both are **negative**, the quotient will be **positive**.

Rule1: If **both** dividend and divisor have the **same sign**, positive or negative, the quotient will be **positive**.

(16) ÷ (4) = 4 (positive quotient).

(-16) ÷ (-4) = 4 (positive quotient).

Rule2: If **either** of the dividend or divisor is **negative** the quotient will be **negative**.

(16) ÷ (-4) = -4 (negative quotient).

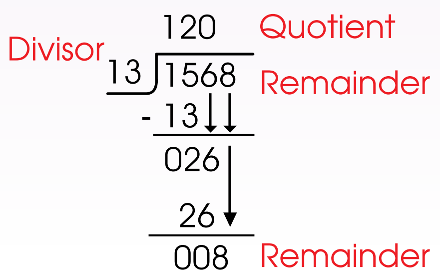
(-16) ÷ (4) = -4 (negative quotient).

In other words (**if the signs are the same, the quotient will be positive, if they are different, the quotient will be negative**).

This rule is for multiplication and division.

Look at this example carefully:

-1568 ÷ 13 = ... Use a calculator to find the answer. You will get -120.615. What does this mean? The answer has a decimal which indicates that the quotient is not a whole number and it's hard to solve mentally. So we need to use long division. Don't use the negative sign within the operation, but put it as the last step.



So the answer is 120 and rem = 8, but the dividend is negative, so the answer is -120 with rem = 8.

Exercises

1. Find the following.

ⓐ 15 + (-11) = 4

ⓑ (-20) + (-32) = -52

ⓒ 64 - 99 = -35

ⓓ -340 - (-20) = -320

ⓔ 10 + (-4) = 6

ⓕ (-3) + (-9) = -12

ⓖ -6 - (7) = -13

ⓗ 34 - (-4) = 38

ⓙ 324 - (-12) + 14 = 350

2. Find the product.

ⓐ 73 x 21 = 1533

ⓑ 40 x (-37) = -1480

ⓒ 3461 x (-10) = -34610

ⓓ (-297) x (-6) = 1782

ⓔ (-418) x (-86) = 35948

ⓕ (-290) x 20 = -5800

3. Find the quotient using long division.

ⓐ $\begin{array}{r} 679 \\ 8\overline{)5432} \end{array}$

ⓑ $\begin{array}{r} 13 \\ 56\overline{)728} \end{array}$

ⓒ $\begin{array}{r} 3 \\ 45\overline{)135} \end{array}$

ⓓ (-216) ÷ (29) =

$\begin{array}{r} 007 \\ 29\overline{)216} \end{array}$

reminder = 13

ⓔ (-896) ÷ (-54) =

$\begin{array}{r} 016 \\ 54\overline{)896} \end{array}$

reminder = 32

ⓕ 659 ÷ (-73) =

$\begin{array}{r} 009 \\ 73\overline{)659} \end{array}$

reminder = 2

1.6 BOMDAS

When a statement contains more than one operation, always solve the operations in this order: Brackets, Multiplication, Division, Addition, Subtraction. Why do you think we call this order BODMAS?

For example: 10 x (20 -5) + 4.
First: Work out the brackets. 20 - 5 = 15.
so we get 10 x 15 + 4.
Second: Work out the multiplication 10 x 15 = 150.
Third: Work out the addition 150 + 4 = 154.
So:

$$10 \text{ x } \underbrace{(20 - 5)}_{\textcircled{1}} + 4$$

$$\downarrow \qquad \qquad \downarrow$$

$$\underbrace{10 \text{ x } 15}_{\textcircled{2}} + 4$$

$$\underbrace{150 + 4}_{\textcircled{3}}$$

$$154$$

Exercise

Work these out without using a calculator.

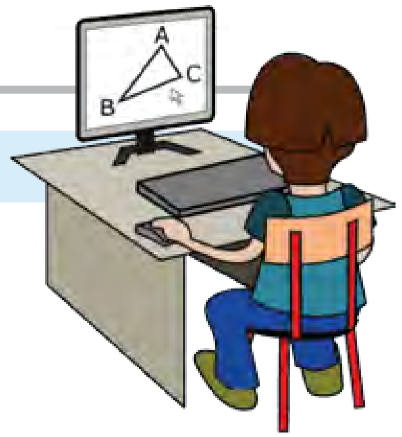
ⓐ 70 - 20 x 2 = 30

ⓑ 21 - (16 - 20) ÷ 2 = 23

ⓒ 12 x (3 + 17) ÷ 2 + 1 = 121

ⓓ $(100 \div \frac{1}{3}) \div (6 + 4) =$ 30

1.7 Problem Solving

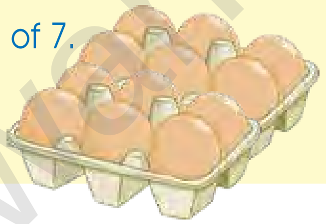


1- Understand the problem.
2- Devise a plan.
3- Carry out the plan.
4- Look back.

1- A farmer sold 500 boxes of oranges, 1711 boxes of apples, 1202 boxes of mangoes and 777 boxes of lime.
How many boxes did the farmer sell altogether?

500 + 1711 + 1202 + 777 = 4190 boxes

2- A box contains 13 rows of eggs with 14 eggs in each row.
The shopkeeper rearranges the eggs in a different box, in rows of 7.
How many eggs are now in each row? 26



3- Work these out.

a) -122 ÷ -33 = 3.69

b) 36 – 24 ÷ 10 X 2 = 31.2

c) 18 + X (10-12) = SKIP

d) 53 – (2+ (0.25 – 1) = 51.75

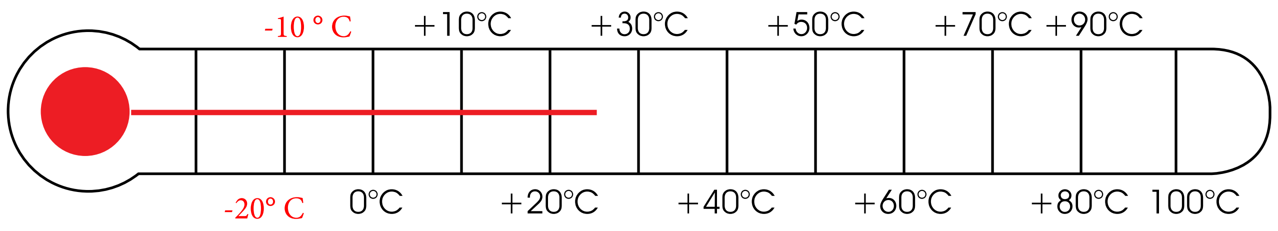
4- Christmas balls arrived in a big box. The box had 48 rows, with 16 balls in each.

a) How many balls were there altogether? 14 * 16 = 768

b) The shopkeeper packaged boxes of balls.
How many boxes did she make, and how many balls were left over? SKIP

5- If a car moves forward, it travels a positive distance, and if it moves in reverse, it travels a negative distance. The car starts from zero (0) and moves forward 5 metres.
Then, it moves 8 metres in reverse. Demonstrate the car movements on a number line. use grid paper

6- The centigrade thermometer is marked for temperatures above the freezing point of water. Complete the marking on the thermometer below the freezing point of water.



2.1 The Simplification of Fractions

A fraction is described as a part of a whole. The number on the bottom of the fraction is called the **denominator**. The number on the top of the fraction is called the **numerator**. In the fraction $\frac{3}{4}$, 3 is the numerator and 4 is the denominator.

① Proper Fractions

Are fractions whose numerator is smaller than the denominator.

$\frac{1}{2}, \frac{3}{4}, \frac{2}{5}$

② Improper Fractions

Are fractions whose numerator is equal to or greater than the denominator.

$\frac{3}{3}, \frac{3}{2}, \frac{12}{5}$

Equivalent Fractions

Have the same value, even though they may look different.

We can find equivalent fractions by **multiplying** or **dividing** the numerator and the denominator of the fraction by the **same number**.

by multiplying:

$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ we see that $\frac{1}{2} = \frac{2}{4}$

by dividing:

$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ we see that $\frac{6}{8} = \frac{3}{4}$

Exercise

Complete the equivalent fractions.

a $\frac{4}{8} = \frac{12}{\boxed{24}}$

b $\frac{6}{21} = \frac{\boxed{18}}{63}$

c $\frac{\boxed{18}}{27} = \frac{6}{9}$

d $\frac{-12}{\boxed{32}} = \frac{-3}{8}$

Simplifying a Fraction

This means that there is no number, except for (1), that can be divided evenly into both the numerator and the denominator to reduce a fraction to its lowest terms. To simplify a fraction, we divide the numerator and denominator by their greatest common factor (GCF).

To find a fraction in its simplest form, we can reduce it to the lowest terms using the following method:

To write $\frac{20}{25}$ in its simplest form:

Find the GCF of 20 and 25.
Factors of 20 : 1, 2, 4, **5**,10, 20
Factors of 25 : 1, **5**, 25
GCF = 5.
Divide the numerator and the denominator by the GCF

$\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$

The simplest form of $\frac{20}{25} = \frac{4}{5}$

Exercise

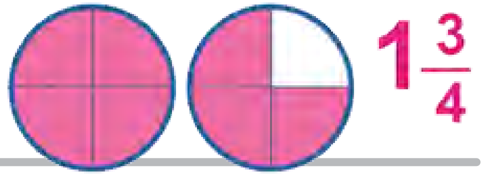
Reduce the following to their simplest form.

a $\frac{6}{18} = \frac{\boxed{1}}{\boxed{3}}$

b $\frac{8}{12} = \frac{\boxed{2}}{\boxed{3}}$

c $\frac{9}{12} = \frac{\boxed{3}}{\boxed{4}}$

d $\frac{32}{120} = \frac{\boxed{4}}{\boxed{15}}$



2.2 Mixed Numbers

Numbers such as $6\frac{2}{3}$ are called mixed numbers.

To Change a Mixed Number into a Fraction

Multiply the whole number by the denominator and add the numerator to the product keeping the denominator without changing.

e.g. $$6\frac{2}{3} = \frac{(6 \times 3) + 2}{3} = \frac{18 + 2}{3} = \frac{20}{3}$$

To Change an Improper Fraction Into a Mixed Number

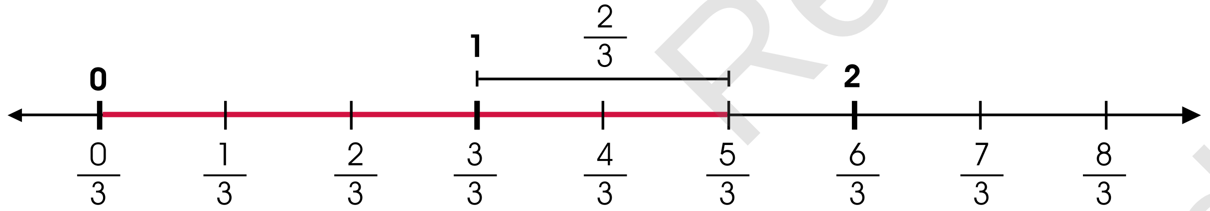
e.g. Divide the numerator by the denominator.
Write the remainder over the divisor.

$$\frac{13}{3} = \begin{array}{r} 4 \\ 3\overline{)13} \\ -\underline{12} \\ 1 \end{array} = 4\frac{1}{3}$$

Remainder

Fractions on the Number Line

We can always find a fraction between two whole numbers by using a number line.



$$\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

Exercises

① Change the following mixed numbers into improper fractions:

ⓐ $3\frac{2}{9} =$ $\frac{29}{9}$ ______

ⓑ $11\frac{1}{9} =$ $\frac{100}{9}$ ______

ⓒ $4\frac{2}{3} =$ 14 ______

ⓓ $16\frac{3}{4} =$ $\frac{67}{4}$ ______

② Change the following improper fractions into mixed numbers:

ⓐ $\frac{53}{4} = 13\frac{1}{4}$

ⓑ $\frac{25}{7} = 3\frac{4}{7}$

ⓒ $\frac{48}{10} = 4\frac{8}{10}$

ⓓ $\frac{36}{8} = 4\frac{4}{8}$

$\frac{2}{5} ? \frac{4}{7}$

2.3 Comparing Fractions

To compare fractions, we rewrite them as equivalent fractions with the same denominators. **If the denominators are the same, the fraction, with the greater numerator is the greater fraction.**

e.g. To find the greater fraction between $\frac{3}{4}$ and $\frac{2}{4}$, two fractions with the same denominator, we look at the numerators. We will find that $\frac{3}{4}$ is greater than $\frac{2}{4}$ because the numerator of the first fraction is greater.

$$\frac{3}{4} > \frac{2}{4}$$

e.g. To find the greater fraction in $\frac{3}{4}$ and $\frac{2}{3}$, two fractions with different denominators, change them into similar fractions , which have the same denominator.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Therefore, $\frac{3}{4}$ is greater than $\frac{2}{3}$. $\frac{3}{4} > \frac{2}{3}$

Mixed numbers can be compared in the same way. First, compare the whole numbers. If they are the same, compare the fractions.

e.g. $2\frac{1}{2} \square 2\frac{1}{5}$

Here, the whole numbers are equal.
By changing the different fractions into similar fractions,

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} \qquad \frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

We can see that $\frac{1}{2}$ is greater than $\frac{1}{5}$.
Therefore, $2\frac{1}{2}$ is greater than $2\frac{1}{5}$.

$$2\frac{1}{5} < 2\frac{1}{2}$$

Exercise

Use > or < or = to make the following statements true.

ⓐ $\frac{-3}{4} < \frac{-2}{10}$ ⓑ $\frac{2}{5} < \frac{7}{12}$ ⓒ $\frac{17}{31} > \frac{5}{14}$ ⓓ $\frac{-11}{20} < \frac{7}{12}$

2.4 Operations With Fractions

Adding and Subtracting Fractions

Be sure that you have the **same denominator** when **adding** and **subtracting**. Then add or subtract the numerator.
You may need to simplify your answer after you combine the two fractions

e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$. Notice that you have the same denominator.

We just need to add the numerators 5 + 1 = 6, and the denominators remain the same (7).

e.g. $\frac{3}{4} - \frac{1}{2} =$. Notice that you have to make the denominators equal, so multiply the second fraction by 2.
Now, you can make the operation $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

Exercise

Solve each problem. Write your answer as a mixed number (if possible).

ⓐ $\frac{4}{7} - \frac{13}{2} = \frac{8}{14} - \frac{182}{14} = \frac{-174}{14} = -12\frac{6}{14}$

ⓑ $\frac{-5}{14} - \frac{-9}{7} = \frac{13}{14}$

ⓒ $\frac{4}{7} + 4\frac{4}{5} = 5\frac{13}{35}$

ⓓ $\frac{-12}{25} - \frac{10}{25} + 2\frac{1}{3} = 1\frac{34}{75}$

ⓔ $3\frac{8}{13} - 6\frac{5}{7} - \frac{-1}{2} + \frac{5}{7} = -1\frac{21}{26}$

To Multiply Two Fractions:

① Multiply the numerator by the numerator.

② Multiply the denominator by the denominator $\longrightarrow \frac{a}{c} \times \frac{b}{d} = \frac{a \times b}{c \times d}$
for all real numbers a, b, c, d ($c \neq 0$, $d \neq 0$)

e.g. Multiply $\frac{1}{4} \cdot \frac{4}{9} = \frac{1 \times 7}{4 \times 9} = \frac{7}{36}$

To Divide Two Fractions:

To divide by a fraction, multiply by the reciprocal for all real numbers a, b, c, d ($b \neq 0$, $c \neq d$, $d \neq 0$).

$$\longrightarrow \frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} =$$

$$= \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

- The reciprocal for $\frac{a}{b}$ is $\frac{b}{a}$ and for $\frac{c}{d}$ is $\frac{d}{c}$
- The product of a number and its reciprocal is 1.

Reciprocals

When the product of multiplying two fractions is 1, the numbers are called reciprocals of each other.

e.g. $\frac{2}{3}$ is a reciprocal of $\frac{3}{2}$

because $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$

Exercises

① Copy to your textbook then add:

- ⓐ $\frac{1}{3} + \frac{2}{3}$ 1
- ⓑ $\frac{1}{2} + \frac{5}{10}$ 1
- ⓒ $\frac{11}{16} + \frac{2}{4} + \frac{7}{8}$ $\frac{33}{16}$
- ⓓ $9\frac{3}{8} + 3\frac{4}{16}$ $\frac{109}{8}$

② Copy to your textbook then subtract:

- ⓐ $\frac{9}{16} - \frac{7}{16}$ $\frac{2}{16}$
- ⓑ $\frac{7}{9} - \frac{1}{3}$ $\frac{2}{27}$
- ⓒ $11\frac{3}{4} - 5\frac{2}{3}$ $48\frac{3}{4}$
- ⓓ $3\frac{7}{10} - 2\frac{1}{2}$ $5\frac{19}{21}$

③ Copy to your textbook then multiply:

- ⓐ $\frac{9}{16} \times \frac{8}{15}$ $\frac{3}{10}$
- ⓑ $\frac{5}{6} \times \frac{18}{75} \times \frac{10}{27}$ $\frac{2}{27}$
- ⓒ $9\frac{3}{8} \times 5\frac{1}{5}$ $48\frac{3}{4}$
- ⓓ $7\frac{1}{9} \times 1\frac{15}{16} \times \frac{3}{7}$ $5\frac{19}{21}$

④ Copy to your textbook then write the reciprocal of the following:

- ⓐ $\frac{7}{8}$ $\frac{8}{7}$
- ⓑ $6\frac{7}{9}$ $\frac{9}{61}$
- ⓒ $4\frac{1}{8}$ $\frac{8}{33}$
- ⓓ $\frac{15}{4}$ $\frac{4}{15}$

⑤ Copy to your textbook then divide the following:

- ⓐ $\frac{3}{8} \div \frac{1}{4}$ $\frac{3}{2}$
- ⓑ $9 \div \frac{1}{6}$ 54
- ⓒ $\frac{8}{3} \div \frac{4}{5}$ $\frac{10}{3}$
- ⓓ $\frac{5}{8} \div 7$ $\frac{5}{56}$

2.5 Property

The Distributive Property of Multiplication Over Addition

$$\frac{1}{2}\left(\frac{3}{4}+\frac{1}{4}\right)=\frac{1}{2}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{4}$$

The example above shows the that multiplication is distributed over addition, in the same way the multiplication is distributed over subtraction.

e.g. Simplify $\frac{1}{4}\left(\frac{1}{2}+\frac{1}{3}\right)$ using the distributive property of multiplication over addition and subtraction.

$$\frac{1}{4}\left(\frac{1}{2}+\frac{1}{3}\right)$$

$$=\left(\frac{1}{4}\times\frac{1}{2}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)$$

$$=\frac{1}{8}+\frac{1}{12}$$

$$=\frac{3+2}{24}$$

$$=\frac{5}{24}$$

e.g. Taking any three fractions, verify the distributive property of multiplication over subtraction.

Let the three fractions be $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. The distributive property of multiplication over subtraction is:

$$\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)$$

$$=\frac{1}{2}\times\frac{1}{3}-\frac{1}{2}\times\frac{1}{4}$$

$$=\frac{1}{6}-\frac{1}{8}$$

$$=\frac{4-3}{24}$$

$$=\frac{1}{24}$$

and $\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)$

$$=\frac{1}{2}\left(\frac{4-3}{12}\right)$$

$$=\frac{1}{2}\times\frac{1}{12}$$

$$=\frac{1}{24}$$

Hence, $\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{2}\times\frac{1}{3}-\frac{1}{2}\times\frac{1}{4}$

Exercises

① Simplify by using the distributive property of multiplication over addition or subtraction.

ⓐ $\frac{2}{5}\left(\frac{3}{4}+\frac{1}{4}\right)$

$\frac{2}{5}$

ⓒ $\frac{6}{7}\left(\frac{2}{9}-\frac{1}{18}\right)$

$\frac{1}{7}$

ⓑ $2\frac{3}{4}\left(\frac{4}{9}+\frac{1}{3}\right)$

$\frac{77}{36}$

ⓓ $1\frac{1}{6}\left(\frac{5}{6}-\frac{2}{9}\right)$

$\frac{77}{108}$

② Verify the following:

ⓐ $\left(\frac{7}{8}-\frac{1}{2}\right)\frac{2}{3}=\frac{7}{8}\times\frac{2}{3}-\frac{1}{2}\times\frac{2}{3}$

This is the distributive property over subtraction.

ⓑ $\frac{3}{8}\left(\frac{2}{5}+\frac{2}{15}\right)=\frac{3}{8}\times\frac{2}{5}+\frac{3}{8}\times\frac{2}{15}$

This is the distributive property over addition.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

2.6 Operations With Decimals

Decimals are a simple way of writing fractions whose denominator is a power of 10.

$\frac{5}{10}$ can be written as 0.5	$\frac{5}{100}$ as 0.05	$\frac{5}{1000}$ as 0.005

The dot (.) is called the decimal point.

Note: *The number of digits to the right of the decimal point is equal to the number of zeros in the denominator of the given fraction.*

The numbers to the left of the decimal point represent whole numbers.
The numbers to the right of the decimal point represent fractions.
The chart below shows the place value of digits to the right of the decimal point:

Note: *Each place has a value of $\frac{1}{10}$ to that of the place to its left.*

Decimal Equivalents

Tenths	$\frac{1}{10} = 0.1$	Hundredths	$\frac{1}{100} = 0.01$
Thousandths	$\frac{1}{1000} = 0.001$	Ten thousandths	$\frac{1}{10,000} = 0.0001$

To read .01 we say one-hundredths or point zero one.

1- Addition and Subtraction of Decimals

To add (or subtract) decimals follow these steps.

1. Write the given numbers one above the other with the decimal points in a vertical line.
2. Add or subtract the numbers as needed.
3. Write the sum or difference.

e.g. Add 21.783 + 1.92 + 32.1

$$\begin{array}{r} 21.783 \\ 1.920 \\ +\ 32.100 \\ \hline 55.803 \end{array}$$

e.g. Subtract 18.71 - 0.3

$$\begin{array}{r} 18.71 \\ -\ 0.30 \\ \hline 18.41 \end{array}$$

Exercises

① Copy to your textbook then add or subtract the following.

ⓐ 46.342 +15.296 61.638	ⓑ 129.235 + 16.41 145.645
ⓒ 24.37 – 18.26 6.11	ⓓ 76.423 – 38.9 37.523
ⓔ 15.003 – 14.386 0.617	ⓕ 49 – 3.276 45.724

② Copy to your textbook then simplify and find the answer.

ⓐ 75.2 + 18.6 – 65.3 28.5	ⓑ 39 – 16.28 + 15.75 38.47

2- Multiplication of Decimals

A- Multiplying by10, 100 and 1000 (Powers of 10)

Steps:

- Move the decimal point to the right as many places as there are zeros in the fraction.
- Move the decimal point one step to the right (10 has one zero).
- Move the decimal point two steps to the right (100 has two zeros).

32.24 x 10 $= \longrightarrow$ 32.24 322.4

The decimal shifts 1 place to the right.

b- Multiplying Decimals by a Whole Number and Multiplying Decimals by Decimals

Steps:

1) Ignore the decimal point.
2) Multiply normally.
3) Put the decimal point in the answer.

It will have as many decimal places of the two original numbers combined.

e.g. 79 x 2.3 =

```
   79
 X 23   <------ 1 place
  237
 158  +
 181.7  <------ 1 place
```

Then we have one place ⟶ 181.7

e.g. 67.3 x 2.7 =

```
   67.3  <------ 1 place
 x  2.7  <------ 1 place
  4711
 1346
 181.71  <------ 2 places
```

Then we have one place ⟶ 181.71

Exercises

① Multiply the following decimals by 10, 100 and 1000. Use your textbook.

ⓐ 7.3 73 730 7300

ⓑ 2.5974 25.974 259.74 2597.4

ⓒ 4.032 40.32 403.2 4032

② Multiply and place the decimal point in the product. Use your textbook.

ⓐ 95 x 5.7 = 541.5

ⓑ 9.32 x 0.26 = 2.4232

ⓒ 7.61 x 1.5 = 11.415

ⓓ 0.68 x 0.47 = 0.3196

3- Division of Decimals

● Division of Decimals by Whole Numbers

Steps:

1- Divide in the same way as with whole numbers.

2- Put a decimal point in the quotient, directly above the decimal point in the dividend.

e.g. 15.48 ÷ 4

```
     3.87
  4⟌15.48
    12↓ |
    ----|
     34 |
   - 32 ↓
    -----
      28
      28
    -----
       0
```

To Check

15.48	÷	4	=	3.87
(dividend)		(divisor)		(quotient)

Multiply the quotient by the divisor.
The product will be the dividend.

3.87 x 4 = 15.48

● Dividing by Powers of 10

When dividing by powers of 10, move the decimal point to the left of the same number of places as the exponent in the divisor.

e.g. When dividing by 10, move the decimal point one place to the left.

130 ÷ 10 = 13
130 ÷ 100 = 1.3

e.g. 4629.2 ÷ 100 = 46.292
Two places to the left

Dividing Decimals by Decimals

To divide decimals by decimals:

1- If the divisor is not a whole number, move the decimal point to the right to make it a whole number. Then, move the decimal point in the answer the same number of places.

2- Divide as usual......

3- Put the decimal point directly above in the dividend.

4- Check your answer.

e.g.

10.14 ÷ 2.6

(1) 2.6 x 10 = 26

(2) 10.14 x 10 = 101.4

$$\begin{array}{r} 3.9 \\ 26\overline{)101.4} \\ -78 \\ \hline 234 \\ 234 \\ \hline 0 \end{array}$$

Exercises

① Divide:

a) 3.52 ÷ 8 — 0.44

b) 26.5 ÷ 5.3 — 5

c) 0.448 ÷ 8 — 0.056

d) 0.888 ÷ 0.444 — 2

② Divide the following by 10,100 and 1000.

a) 4.6 — 0.46
0.046
0.0046

b) 459.32 — 45.932
4.5932
0.45932

c) 64.583 — 6.4583
0.64583
0.064583

③ Multiply quickly:

a) 3.165 x 10 — 31.65

b) 12.74 x 100 — 1274

c) 3.654 x 1000 — 3654

④ Divide quickly:

a) 2.745 by 1000 — 0.002745

b) 2.745 by 100 — 0.02745

c) 13.26 by 10000 — 0.001326

2.7 Converting Between Fractions and Decimals

Converting Fractions to Decimals

Method 1

Look at the fraction, then try to find the number, which if we multiply the fraction by, we get 10 or 100 or 1000 .. etc. Let's try:

$$\frac{4}{25} = \frac{4 \times 4}{25 \times 4} = \frac{16}{100} = 0.16$$

Method 2

Divide directly $\frac{1}{4} = 1 \div 4$

$$\begin{array}{r} .25 \\ 4\overline{)1,0} \\ -\ 8 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$\frac{1}{4} = 0.25$

Exercise

1- Copy to your notebook then convert the following fractions into decimals.

a $\frac{2}{5}$ b $\frac{3}{4}$ c $\frac{9}{100}$ d $\frac{45}{200}$ e $\frac{24}{25}$ f $\frac{11}{20}$ g $\frac{6}{13}$

h $\frac{1}{19}$ i $\frac{7}{18}$ j $\frac{4}{15}$ k $\frac{19}{20}$

Converting Decimals to Fractions.

Steps:

① Write down the decimal divided by 1 like this: $\frac{\text{decimal}}{1}$

② Multiply both top and bottom by 10 for every digit to the right of the decimal point. (For example, if there are two numbers after the decimal point, then use 100, if there are three then use 1000, etc).

③ Simplify the fraction.

e.g. Write down 0.75:

$$\frac{0.75}{1}$$

$$\frac{0.75 \times 100}{1 \times 100} = \frac{75}{100}$$

$$\frac{75}{100} \curvearrowright \frac{15}{20} \curvearrowright \frac{3}{4}$$

e.g. Change the following decimal fractions into common fractions:

ⓐ $1.5 = 1\frac{5}{10} = \frac{15}{10} = \frac{3}{2} \quad 1.5 = \frac{3}{2}$

ⓑ $0.25 = \frac{25}{100} = \frac{1}{4}$

ⓒ $3.25 = 3\frac{25}{100} = \frac{325}{100} = \frac{13}{4} = 3\frac{1}{4}$

Exercises

① Convert the following into fractions.

ⓐ 0.3 $\frac{3}{10}$	ⓑ 0.75 $\frac{3}{4}$	ⓒ 6.25 $6\frac{1}{4}$	ⓓ 0.25 $\frac{1}{4}$
ⓔ 1.875 $1\frac{7}{8}$	ⓕ 0.0005 $\frac{1}{2000}$	ⓖ 0.075 $\frac{3}{40}$	ⓗ 0.675 $\frac{27}{40}$

② Write each of the following as a decimal fraction.

ⓐ 1 tenth 0.1	ⓑ 5 hundredths 0.05	ⓒ 3 thousandths 0.003

③ Arrange in descending order.

ⓐ -1.37 , -1.74 , -1.26 , -1.093 -1.093 , -1.26 , -1.37 , -1.74	ⓑ $\frac{4}{5}$, $\frac{5}{8}$, 0.75 , 0.66 4/5 , 0.75 , 0.66 , 5/8

④ Arrange in ascending order.

ⓐ 0.33 , 0.23 , 0.04 , 0.008 0.008 , 0.04 , 0.23 ,0.33	ⓑ $\frac{3}{-16}$, -0.281, -0.099 , $\frac{1}{-8}$ -0.281 , 3/-16 , 1/-8 , -0.099

Uses of Brackets Involving Arithmetic Operations

BODMAS :

Bodmas is an abbreviation for **(B)rackets**, **(O)rder**, **(D)ivision**, **(M)ultiplication**, **(A)ddition** and **(S)ubtraction**. Complicated expressions can be simplified in the following way:

Step: ① — Brackets-First. Handle the expressions within the brackets, following the sequence:

1 bar → 2 parentheses → 3 braces → 4 brackets,

Step: ② — Follow by → Division → Multiplication.

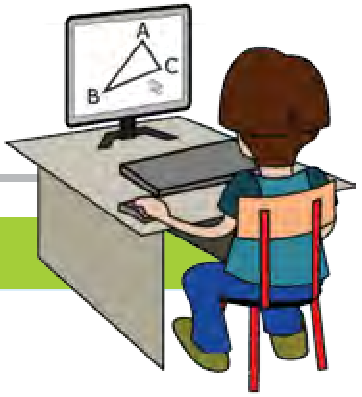
Step: ③ — Follow by Addition → Subtraction.

Exercise

Simplify the following:

ⓐ $\frac{1}{2} + (\frac{1}{3} + \frac{1}{4})$ 13/12	ⓑ $\frac{4}{3} \times (\frac{4}{3} - \frac{1}{3} + \frac{1}{2})$ 2
ⓒ $(\frac{1}{6} \div \frac{3}{4}) \div (\frac{3}{8} \div \frac{1}{4})$ 4/27	ⓓ $1\frac{1}{5} - (\frac{2}{3} - \frac{1}{3} + \frac{1}{5}) \times \frac{4}{9}$ 26/27
ⓔ $(6 + 3 \times (4 + 5)) \times 7 + 8 \times 5$ 271	ⓕ $2 \times (3.5 - 2.05) + 6 \times (1.5 - 1.4)$ 7/2
ⓖ $8.21 + (6.01 \div (1.38 - 0.99))$ 23.62	ⓗ $(77 - (16 + 11) \div 5) + 18$ 448/5
ⓘ $0.3 \times 1.3 + (0.5 + (1.2 - 5.1 - 5.01))$ -401/50	ⓙ $(\frac{1}{3} + (\frac{5}{6} - \frac{3}{4})) + (\frac{5}{6} - (\frac{1}{2} - \frac{1}{6}))$ 11/12

2.8 Problem Solving



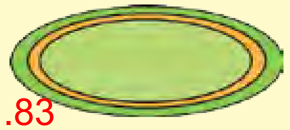
1- Understand the problem.
2- Devise a plan.
3- Carry out the plan.
4- Look back.

- **Solve the following word problems.**

1- There is 0.625 kg of powdered milk in each tin. If a carton contains 12 tins, find the total mass of powdered milk in the carton. 0.625 * 12 = 7.5

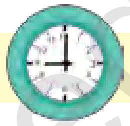
2- The mass of a jar of sweets is 1.6 kg. What is the total mass of 8 such jars of sweets? 1.6 * 8 = 12.8

3- Carpeting costs $9.99 a yard. If Jane buys 17.2 yards, how much will it cost her?
(Round your answer to the nearest hundredth)9.99 * 17.2 = 171.83



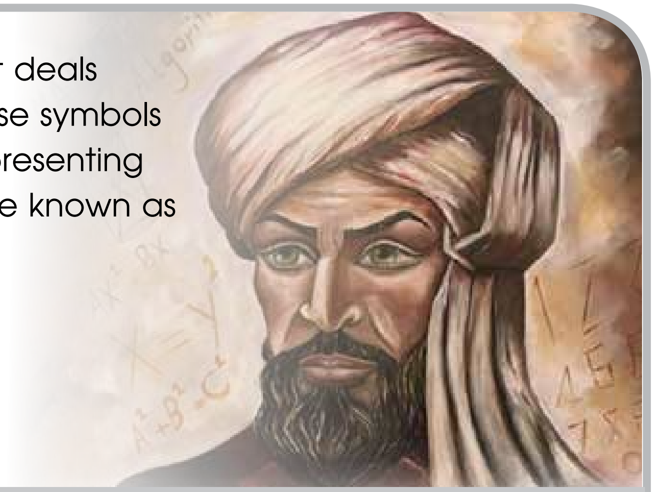
4- Order from least to greatest: $\frac{8}{9}$, $\frac{17}{18}$, $\frac{2}{3}$, $\frac{7}{6}$. 2/3 , 8/9 , 17/18 , 7/6

5- What fraction of 1 hour is 50 minutes?



Algebra

Unit 3



Algebra is a branch of mathematics that deals with symbols. In elementary algebra, those symbols are written as Latin and Greek letters representing quantities without fixed value, and they're known as variables.

Who first founded algebra?

Review

The order of operations is:
1- Simplify the expressions inside parentheses ( ) and brackets [ ].
2- Evaluate all powers.
3- Do all multiplications and divisions from left to right.
4- Do all additions and subtractions from left to right.

e.g. 4 x 8 - 2 =

We have to do the multiplication first. Then, we continue with subtracting 32 - 2 to get 30.

Exercise

Simplify the following expressions:

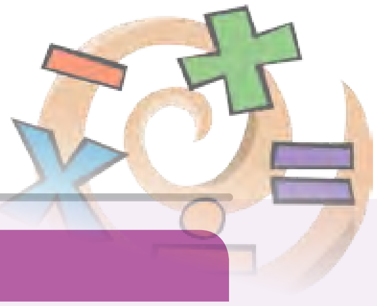
a $(2 + 3) \times (7 - 3)^2 =$ 80

b $4 + 5 \times \frac{3}{5} =$ 7

c $36 + 15\,(40 - 35) =$ 111

d $2 \times \left((8 - 3)^2 + 6 \times 2\right) =$ 74

$$\underbrace{3x^{\overset{\text{exponent (power)}}{2}}}_{\text{term}} - \underbrace{2xy}_{\text{term}} \underset{\text{operation}}{+} \overset{\text{constant}}{c}$$



3.1 Algebraic Expressions

An algebraic expression is the combination of variables, constants, coefficients, exponents, and terms which are combined together by the following arithmetic operations: addition, subtraction, multiplication, and division. An example is given below:

$3x + 7y$ - 10. Read it and give another example.

Variables: Alphabetical charcters which are used for assigning the value. Widely used variables are x, y, t, k ....

Constant: Algebraic constants have values that never change like

2, 15, $\frac{-2}{3}$ ....

Term: Term of the algebraic expression is concatenated to form the algebraic expression by arithmetic operations such as addition, subtraction, multiplication and division. For example:

2x + 10

The number that multiplies by a variable is generally written in front it. For example:

2x, the coefficient is 2.

When the coefficient equals 1, it is usually is not written.

(i.e. ? $1xy = xy$, $x = 1 \times x$).

Exercise: Complete the following table.

Algebraic expression	Number of terms	Terms	Variables	Coefficient
$2x + 9$	2	$2x$ 9	x	2
$2x^2 + 3y + 7$	3	$2x^2$ $3y$ 7	x , y	2, 3
$7x^2 - 12by^2$	2	$7x^2$, $12by^2$	x, by	7, 12
$\frac{9}{12} + \frac{7b^2x}{8} - 3y$	3	9/12, $7b^2x/8$, 3y	b, x, y	7, 1, 3

In other words:

A variable is a letter used to represent a value that can change.

$\boldsymbol{x}$

A constant is a value that does not change.

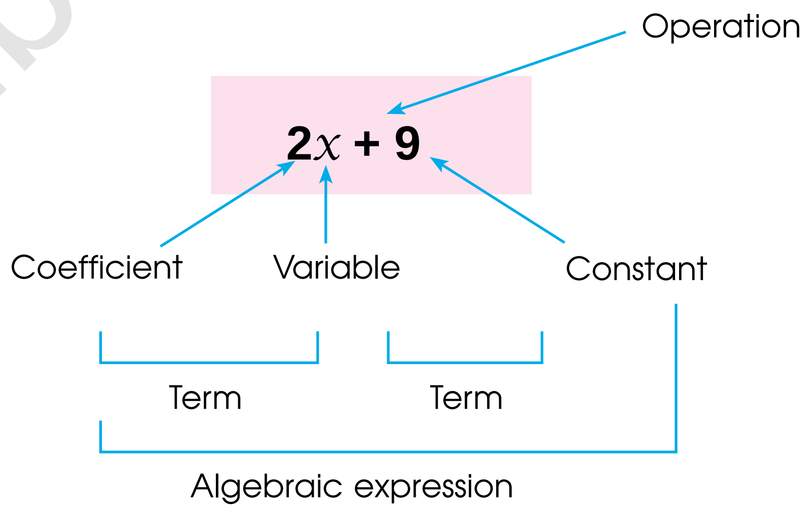
9

A numerical expression contains only constants and operations.

2 + 9

An algebraic expression may contain variables and constants.

$2x$ + 9



3.2 Combining Like Terms

Simplify an expression by combining like terms. For example $3a + 6b - a + 3b$.

Combine the like terms $3a$, $(-a)$ to get $3a + (-a) = 2a$ and combine the like terms $6b$, $3b$ to get $6b + 3b = 9b$.
Then 3a + 6b - a + 3b is simplified to 2a + 9b.

e.g. ⓐ $10k^2 + 15k^2 = 25k^2$

ⓑ $7xy - 6xy + 7x = xy + 7x$

Algebraic expressions	Simplest form
$3t+12y-t+10a-10t+5y+t$	$10a-7t+17y$
$7x^2+y^2-6x^2+y$	$x^2 + 2y$
$xy+3x+2y+xy-x-y$	$2xy + 2x + y$
$14b-2b^2+4b$	$18b - 2b^2$
$26+5h-10+12k-5b-12k-6$	$10 + 5h - 5b$
$25t^3+14t^3-8d^2-10+2d^3$	$39t^2 - 8d^2 - b + 2d^3$

Complete the following table.

Do you notice that:

$x+y \neq xy$

$x+y \neq 2x$ or $2y$

$x^2+x \neq x^3$ or $2x^2$ or $2x$ or $2x^3$

Exercises

1- Simplify the following expressions.

① $10y - 12y + 6$ $= 6 - 2y$

② $2x^2 - 3x - 2x^2$ $= -3x$

③ $9t^2 - 7t + 4 - 2t^2 + 5t$ $= 7t^2 - 2t + 4$

④ $3xy + 3(xy + z)$ $= 6xy + 3z$

Remember, distributive priority: $a(b + c) = ab + ac$

2- Simplify the following expressions.

- The expression for the operation described:

a) x multiplied by 5 is: $5x$

b) x plus 5 is: $x + 5$

c) 3 times v is: $3v$

d) The sum of v and x is: $v + x$

e) 6 divided by 6 is: 1

f) The quotient of 4 and x is: ____

g) 4 decreased by y is: $4 - y$

h) 6 increased by g is: $6 + g$

i) Raise 3 to x then multiply f by the result: ____

3- Write an expression that represents the statement of the following:

a) John ate x out of 34 cookies: 34 - X

b) Sara bought 19 chocolates, Mike bought x times as many chocolates as Sara, write an expression that shows how many chocolates Sara bought: SKIP

c) Paul has x red peppers and 20 green peppers, write an expression that shows how many peppers Paul has: X + 20

3.3 Evaluating Variables

Evaluating mostly means simplifying an expression down to a single numerical value.

Evaluate each expression when:
$x = 2, \quad y = 1, \quad z = 0, \quad m = \frac{1}{2}$

① $2x + 3yz = 2(2) + 3(1)(0) \longrightarrow 4 + 0 = 4.$

② $x^2 + 2y - 2m = (2)^2 + 2(1) + 2\left(\frac{1}{2}\right) \longrightarrow$ 7

③ $xy - 3z - xm = (2)(1) - 3(0) + 2\left(\frac{1}{2}\right) \longrightarrow$ 1

To evaluate a variable expression:

① Substitute the given numbers for each variable.

② Use the order of operations to evaluate the algebraic expression.

③ Check!

Exercise

1- Find the value of the following:

$(15b - 5x) + 2(4b) + 3 =$ ______ for $b = -1, x = 2$

1st step: $(15(-1) - 5(2)) + 2(4(-1)) + 3 =$

2nd step: $(-15-10) + 2(-4) + 3 =$

$(-25) + (-8) + 3 =$

$(-33) + 3 =$

30

Check it by calculator, is it right?

2- Find the value of the following expressions.

a $2 + z$ ___18___ for $z = 16$.

b $(70 - v) \div 3$ ___22/3___ for $v = 48$

c $k + (516 - 216)$ ___550___ for $k = 250$

d $756 \div (42 \times c)$ ___9___ for $c = 2$

e $(v + 1) \div 6$ ___-2/3___ for v -5

f $\frac{5}{6} \div g$ ___-1___ for $g = \frac{-5}{6}$

g $i \div 9$ ___0.5___ for $i = 4.5$

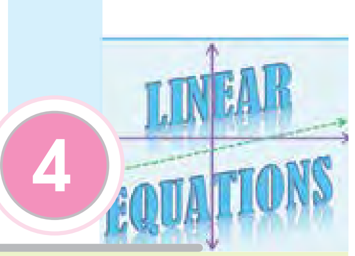
h $m \div (10.6 + 5.4)$ ___0.75___ for $m = 12$

i $2(3 + 2k)$ ___14___ for $k = 2$

j $2 \times (x + 2)$ ___0___ for $x = -2$

Equations and Inequations

Unit 4



4.1 The Equation

Look at these number sentences:

Write (T) when the statement is true, and (F) when the statement is false.

7 + 3 = 10 T

15 -19 = -4 T

7 + 7 = 24 F

15 + 6 = 5 F

20 + 5 = 30 - 5 T

6 + 12 = 30 -12 T

What do you notice?

Lets' take $\underbrace{20 + 5}_{\text{Left side}} = \underbrace{30 - 5}_{\text{Right side}}$ We have two equal sides.

The Equation

is a statement that two things (expressions) are equal.

For example the (expression) 20 + 5 is equal to the (expression) 30 - 5 (because they both equal 25).

Let's make sure we know the difference between an expression and an equation.

Which of these is an equation? (Put ✓).

Exercise

a 8 + 4	b 6 + 3 = 10 - 1 ✓
c Zero.	d $K + 3 = 4$ ✓
e 4 - 2 = 2 ✓	f 3 (10 ÷ 2)

Most equations include a variable. For example, $x + 3 = 4$ has a variable ⓧ If an equation contains a variable, it's called an algebraic equation. For example: $x + 3 = 4$ has a variable, which is x.

An equation introduces two equal things.

$$x + 3 = 4$$

That equation says: What is on the left ($x + 3$) is equal to what is on the right (4). So, an equation is like a statement of "this equals that".

If we substitute x for 0,

we get $0+3 \neq 4$, 3 doesn't equal 4, so $x \neq 0$

Lets try $x = 1$

1+3 = 4. It is a true statement. Can you find more solutions?

Exercises

Choose the answer that makes the equation true.

1 $7 + x = 12$. So $x =$

a 2 = 10 b $x = 9$ [✓ c $x = 5$] d $x = 4$

2 $1 + y = 20$. So $y =$ .....

a $y = 10$ b $y = 1$ c $y = 20$ [✓ d $y = 19$]

3 $2 (x) = 20$. So x = ...

[✓ a $x = 10$] b $x = 2$ c $x = 20$ d $x = 5$

4.2 Solving One-Step Equations

You have already studied equations where the solutions were quite easy to solve by using mental math or patterns, most equations are harder to solve mentally, so they need simplifying before you can get the solution. One way to do this is by:

Using Inverse Operations

The inverse operation of addition is subtraction, and the inverse operation of subtraction is addition.

So: **$x + 4 = 22$**

We have to subtract 4 from both sides.

$x + 4 = 22$
-4 -4

$x = 22 – 4$
$x = 18$

Then, check your answer:
Is (18) + 4 = 22 ?
22 = 22 ✓ So the answer is right.

Otherwise, an inverse operation is an operation that reverses the effect of another operation.
Addition and subtraction are inverse operations.
Division and multiplication are inverse operations too.
Find the value of y in: $2y = 10$
Notice that the operation between 2 and y is multiplication, so the inverse operation is division. Then, divide both sides by 2.

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{10}{2}$$
$$y = \frac{10}{2}$$
$$= 5$$

Check ⟶ 5 (5) = 10
10 = 10

Exercise

Solve each equation, then check your answer.

ⓐ $m - 9 = 20$
m = 29

ⓑ $6z = 6$
z = 1

ⓒ $3 + p = 8$
p = 5

ⓓ $\frac{m}{11} = 7$
m = 77

ⓔ $15 + y = 17$
y = 2

ⓕ $4g = 8 + 12$
g = 5

ⓖ $3 = k - 2$
k = 5

ⓗ $6.3 = v + 2$
v = 4.3

ⓘ $8x - 7x = 12$
x = 12

ⓙ $x - \frac{1}{2} = \frac{3}{4}$
x = 5/4

4.3 Solving Two-or-More-Steps Equations

Equations here take two steps to solve (we have more than one operation).

1. Simplify through using the inverse of addition or subtraction.
2. Simplify through using the inverse of multiplication or division.

Solve

$3x - 10 = 14$

First step ⟶ add 10 to both sides (because we have subtraction).

to get $3x = 14 + 10$

$3x = 24$

Second step ⟶ divide by 3 ⟶ both sides.

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{24}{3}$$

$$x = \frac{24}{3}$$

$$x = 8$$

Now check your answer and substitute ($x = 8$) in the original equation.

⟶ $3\ (8) - 10 \overset{?}{=} 14$

$24 - 10 = 14$

$14 = 14$ So, the answer is right.

Exercise

Solve each equation.

a. $5n + 5 = 45$ n = 8

b. $\frac{y}{6} - 3 = -11$ y = -48

c. $4\ (x - 1) = 24$ x = 7

d. $12 = \frac{g - 14}{2}$ g = 38

e. $\frac{k + 9}{2} = 0$ k = -9

f. $0.4\ x + 2.9 = 1.5$ x = -3.5

g. $14.2 = 9\ (-5 + t)$ t = 6.577

h. $-2\ (10 - x) = -50$ x = -15

4.4 Inequalities

Equations and inequalities are both mathematical sentences formed by relating two expressions to each other in an equation. The two expressions are deemed equal which is shown by the symbol (=).

$x = y$. x is equal to y.

Whereas in an inequality, the two expressions are not necessarily equal which is indicated by symbols ($>$, $<$, $\leqslant$ or $\geqslant$)

$x > y$: x is greater than y.

$x \leqslant y$: x is greater than or equal to y.

$x < y$: x is less than y.

$x \leqslant y$: x is less than or equal to y.

An equation or an inequality that contains at least one variable is called an **open sentence.**

$x > 5$, what is the possible value of x? ______________________

$x + 2 < 8$, what is the possible value of x? ______________________

Example

Look at these inequalities then answer by T or F.

a. $6 + 5 - 5 < 13 - 5$ (T).

b. $\frac{y}{2} + 3 < 5\ (y = 2)$ (T).

c. $4 - \frac{3x}{5} \geqslant 27\ (x = -7)$ (F).

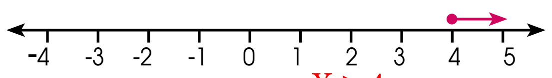
d. $x + 4 > 12$, $x = 13$ (T).

e. $x + 5 < 0$, $x = (-10)$ (T).

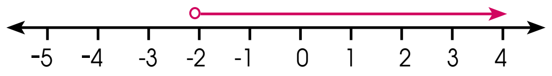
f. $2y - 7 \leqslant 14$, $y = (10)$ (T).

Exercises

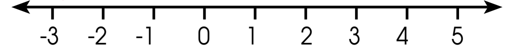
1- Write down the inequalities that represents the following graphs.

a.


X ≥ 4

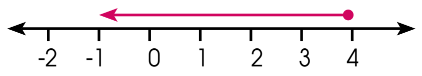
b.


x > -2

c.


x < 1

Before you answer, read the next note.

d.


x ≤ 4

Note: *Use a closed dot (●) to indicate the number itself is part of the solution with $\leq$ and $\geq$.*
Use an open circle (○) to indicate the number itself is not part of solution with < and >.

2- Write the following inequalities through symbols and give examples of solutions.

a A number decreased by 2 is greater than or equals 5. $x - 2 \geq 5$

b Twice a number decrsased by 4 is greater than 8. $2x - 4 > 8$

c Half a certain number increased by 5 is less than ten. $\frac{x}{2} + 5 < 10$

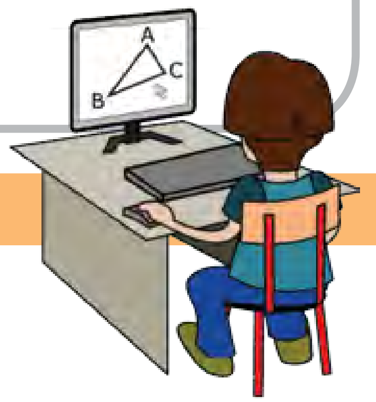
3- Solve the following inequalities:

a $5x - 2 < 8$ $x < 2$

b $x + 7 < 9$ $x < 2$

c $x + 7 < 9$ skip

4.5 Problem Solving



1- Understand the problem.
2- Devise a plan.
3- Carry out the plan.
4- Look back.

1- Sara bought y sweets, she gave 4 sweets to her sister.

(a) How many sweets does she have now? $y - 4$

(b) If she now has 8 sweets, write an equation and use it to find the value of y $y - 4 = 8$
$y = 12$

2- There are x apples on the table. The children ate 6 of them.

(g) How many apples are left? $x - 6$

(b) If x apples are left, write an equation and use it to find what number x stands for. skip

3- Louis weighs k kilograms and his father weighs 70 kilograms.

(a) What is their total weight? $y = k + 70$

(b) If their total weight is 100 kilograms write an equation and use it to find the value of k. $k + 70 = 100$
$k = 30$

4- Write an equation for each of the following and solve it.

(a) 12 less than x equals 15. $x - 12 = 15$
$x = 27$

(b) 7 more than x equals 25. $x + 7 = 25$
$x = 18$

(c) Add 11 to k and the answer is 20. $k + 11 = 20$
$k = 9$

(d) A father is 40 years old, his daughter is x years old, their total age is 52 years.
$x + 40 = 52$
$x = 8$

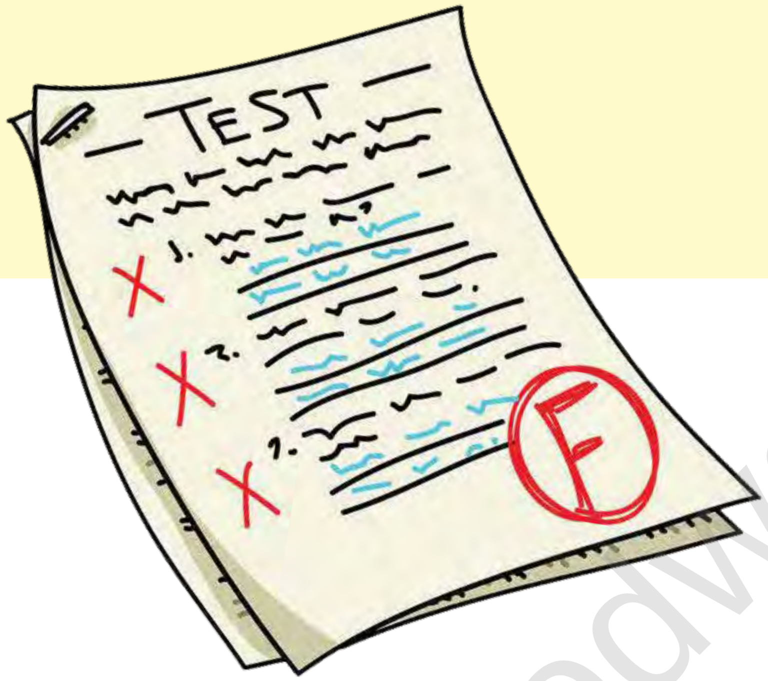
5- Lee answered all the questions on his math test but got 10 answers wrong.

He received 4 points for every correct answer, and there was no penalty for wrong answers.

His score was 76 points.

Write an equation to determine the total number of questions (q) on Tim's math test.

$4q = 76$
$q = 76/4$
$q = 19$



5.1 Ratios

The comparison by division of two quantities in the same unit is a ratio.
The ratio of 2 to 3, is denoted by 2 : 3 (is read 2 to 3) and is measured by a fraction ($\frac{2}{3}$).

e.g.

① The speed of a car is 60 kilometers per hour and the speed of a bus is 45 kilometers per hour. The ratio of their speed is 60 : 45

$= \frac{60}{45} = \frac{4}{5}$ How can you read this ratio ?

② The price of a banana ice cream and a mango ice cream is $2 and $3 respectively. The ratio of their price is

$= 2 : 3$

$= \frac{2}{3}$

Can you get the price of the banana ice cream if the mango ice cream is priced as $9?

Simplify the ratio:

$\frac{2}{4} : \frac{5}{6}$

$= \frac{2}{4} \times \frac{6}{5} = \frac{\cancel{12}^{3}}{\cancel{20}_{5}}$

$= \frac{3}{5} = 3 : 5$

or $2 \times 6 : 4 \times 5$

$= 12 : 20$

$= \frac{12}{20} = \frac{3}{5} = 3 : 5$

Ratios should be expressed as simply as possible.
A ratio is unchanged if the two numbers (or quantities) of the ratio are multiplied and divided by the same number.

Exercises

1 Find the ratios of the following:

a $15 and $20
15 : 30
15/20 = 3/4

b 6 men and 12 men
6 : 12
6/12 = 1/2

c 30 days and 25 days
30 : 25
30/25 = 6/5

2 If Sami's height is 1.60m and Dani's height is 1.75 m, find the ratio of Sami's height to Dani's height.

3 Simplify the following ratios:

a 8 : 12 = 2/3

b 25 : 75 = 1/3

c $\frac{1}{3} : \frac{1}{4} =$ 1/12

d $\frac{3}{8} : \frac{5}{16} =$ 15/128

4 Sally is $1\frac{3}{4}$ m tall and Suzan is $1\frac{1}{2}$ m tall. What is the ratio of Sally's height to Suzan's height? 7/4 * 3/2 = 21/8
21 : 8

5 Express the following as simple ratios.

a 27 kg to 92 kg. 27/92

b 24 cm to 73 cm. 24/73

c 2 hours to $\frac{2}{3}$ hour. 120/40 = 12/4 = 3

d 4.5 litres to 8.5 liters.

5.2 Continued Ratios

Sometimes, we have to compare more than two quantities.

e.g. The ratio between Sami's and Sonny's shares is 2 : 3 and that between Sonny's and Dani's shares is 3 : 4 . What is the ratio between the shares of Sami and Dani?

Here Sonny's share is common. Arrange the ratios as shown below.

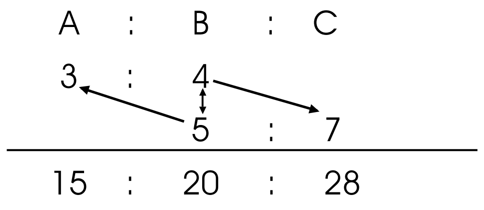
Sami's share	:	Sonny's share	:	Dani's share
2	:	3		
		3	:	4
2	:	3	:	4

Thus, the ratio of the shares of Sami and Dani is 2 : 4 or 1 : 2 (simplified form). This type of comparison is called comparing with the help of continued ratio.

In the example above, Sonny's share is 3 in both ratios. i.e Sonny's share is common.

Now we will take an example where the common share is not the same number.

e.g. The ratio in the ages of A and B is 3 : 4 and of B and C is 5 : 7.
Find the continued ratio among the ages of A, B and C.
Here B is common to both the ratios. Arrange the ratios as shown below, and then multiply as indicated by the arrows to get the continued ratio.



Hence, A : B : C = 15 : 20 : 28

To find the continued ratio between four quantities, we first find the ratio between three quantities and then the ratio between all of the four quantities.

e.g. The ratio in the income of A and B is 2 : 3, and of B and C is 4 : 5 and of C and D is 6 : 5 Find the ratio between the incomes of A, B, C and D.

A	B	C	D
2	3		
	4	5	
8	12	15	
		6	5
48	72	90	75
16	24	30	25

Thus, A : B : C : D = 16 : 24 : 30 : 25 (simplified form)

Note: **In order to reduce the ratio to its simplest form, each number is divided by the common factor.**

Exercises

1. Find the continued ratio between A, B and C when A : B is 4 : 5 and B : C is 5 : 6. 20 : 25 : 30

2. The ratio between the marks obtained by Ali and Jake is 2 : 3 James and Alfred is 5 : 6 while Frank and Shane is 3 : 4 . Find the ratio between the marks obtained by James and Shane. 72

3. 1) A : B = 3 : 5, B : C = 3 : 4 and C : D = 5 : 6 Find A : B : C : D in the simplest form. 9 : 15 : 20 : 24
   2) A : B = 20 : 27, B : C = 8 : 15, C : D = 5 : 6. Find A : B : C : D 100 : 216 : 405 : 486

4. The ratio in the shares of A and B is 5 : 11, B and C is 4 : 5 and C and D is 7 : 8. Find the continued ratio among A, B, C and D. 140 : 308 : 385 : 440

5.3 Proportional Division

Suppose a stick is cut into three pieces - A, B and C- such that A is 6 cm long, B is 9 cm long and C is 12 cm long. We can then say that the ratio of the length of A to the length of B is 6 : 9 or 2 : 3 and the ratio of the length of B to the length of C is 9 : 12 or 3 : 4. We can also say that the stick is cut into pieces in the proportion 2 : 3 : 4. We can write: length A: length B: length C = 2 : 3 : 4

This means that the length of A = $\frac{2}{3}$ of the length of B.

The length of A = $\frac{2}{4}$ or $\frac{1}{2}$ of the length of C.

The length of B = $\frac{3}{4}$ of the length of C.

The length of B = $\frac{3}{9}$ or $\frac{1}{3}$ of the original length of the stick.

The length of A = $\frac{2}{9}$ of the original length of the stick.

To divide a certain quantity in a given ratio is called proportional division.

e.g. Divide$1500 among Farah, Lara and Sara in the ratio of 3 : 4 : 5

Amount = $1500

Ratio = 3 : 4 : 5

Sum of ratios = 3 + 4 + 5 = 12

Farah's share = $\frac{\text{ratio}}{\text{sum of ratios}}$ x amount

= $\frac{3}{12}$ x 1500

= $375

Lara's share = $\frac{4}{12}$ x 1500

= $500

Sara's share = $\frac{5}{12}$ x 1500

= $625

Therefore,

Farah's share = $375

Lara's share = $500

And Sara's share = $625

Note: **Each Share** $= \dfrac{\text{ratio}}{\text{sum of ratios}} \times$ **Amount**

Application to Partnership and Profit Sharing

When two or more people invest in a business, the profit obtained is divided according to their respective investments, the ratio of profit will be the same as the ratio of the investments. If two people invest equally, the profit will be divided equally. If two people invest in the ratios of 1 then it will be divided in the ratios of 1 : 2.

Exercise

Divide $9100 among Ali, Sami and Hillary in the ratio 2 : 4 : 7

e.g.

① Sara invests $100,000 in a business and Selena invests $200,000. At the end of the year, the profit is $9,000. What will their respective shares in the profit be?

Ali = (2/13) * 9100 = 1400
Sami = (4/13) * 9100 = 2800
Hillary = (7/13) * 9100 = 4900

Sarah’s investment	:	Selena's investment
$100,000	:	$200,000

Ratio of investment 1 : 2
Ratio of profit 1 : 2
Sum of ratios = 1 + 2 = 3
Profit per unit ratio (Is one part of the profit out of 3): $\dfrac{9{,}000}{3}$
Sara’s share = $\dfrac{9{,}000}{3} \times 1 = 3{,}000$
Selena's share = $\dfrac{9{,}000}{3} \times 2 = 6{,}000$

e.g. Sami, Sonny and Dani invest in a business, $500,000, $400,000 and $300,000 respectively. They earn a profit of $18,000. Find the shares of each partner. Ratio of investment:

Sami	:	Sonny	:	Dani
$500,000	:	$400,000	:	$300,000
5	:	4	:	3

Sum of ratios : 5 + 4 + 3 = 12

Profit per unit ratio (that is, one part of the profit out of 12): $\dfrac{18{,}000}{12}$

Sami's share $= \dfrac{18{,}000}{12} \times 5 = \$7{,}500$

Sonny's share $= \dfrac{18{,}000}{12} \times 4 = \$6{,}000$

Dani's share $= \dfrac{18{,}000}{12} \times 3 = \$4{,}500$

Exercises

① A class has 30 girls, two–thirds have long hair. Find the number of girls with long hair. 20 = (30/3 * 2)

② A boy has $60 pocket money, he spent two–thirds of it. What amount did he spend? 60/3 * 2 = 210

③ Three fishermen caught 1,380 fishes. If the ratio between their fish is 8 : 9 : 6 How many fish did each of them catch?

④ Divide the given amount between Selena and Linda in the given ratios:

ⓐ $100 in the ratio 2 : 3

100/5 * 2 = 40 (Selena)
100/5 * 3 = 60 (Linda)

ⓑ $4,500 in the ratio $\frac{1}{5} : \frac{1}{4}$

S.O.R = 0.9
4500/0.9 * 0.5 = 2500 (Selena)
4500/0.9 * 0.21 = 2000 (Linda)

5.4 Direct Proportion

An expression of equality between two ratios is called a 'proportion'. The ratios 2 : 3 and 4 : 6 are equal, hence, 2 : 3 = 4 : 6 . This may also be written as

$$2 : 3 :: 4 : 6$$

Look at this example:

Sami earns $20 and saves $5.

Sara earns $16 and saves $4.

Since $\frac{5}{20} = \frac{1}{4}$ and $\frac{4}{16} = \frac{1}{4}$

$$\frac{5}{20} = \frac{4}{16}$$

So the two ratios are equal.

The above proportions may also be written as:

$$5 : 20 :: 4 : 16$$

It is read as 5 is to 20 as 4 is to 16.

The four numbers used in a proportion are the terms of the proportion.

In the above proportion, 5 is the first term, 20 is the second term, 4 is the third term and 16 is the fourth term.

The second and third terms are called the means. The first and the last terms are called extremes.

In the proportion 5 : 20 = 4 : 16, the 20 and the 4 are means and they are also called 2nd and 3rd proportions while the 5 and 16 are extremes, and also called 1st and 4th proportions.

In any proportion the product of means equals the product of extremes.

In the proportion 5 : 20 = 4 : 16

5 x 16 = 80 Product of extremes

20 x 4 = 80 Product of means

If three terms of a proportion are known, the missing term can be found.

e.g. 2 : 4 and 1 : 2 Is it a proportion?

$$2 : 4 :: 1 : 2$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

So, yes, it is a proportion.

A 30-cm-long piece of string costs $5. What is the cost of a 45-cm-long string.

$$\square : 45 :: 5 : 30$$

$$\frac{\square}{45} = \frac{5}{30}$$

$$45 \times \frac{\square}{45} = 45 \times \frac{5}{30}$$

$$\square = \$7.50$$

Note: All the examples given above are examples of direct proportions. In a direct proportion, two quantities are related in such a way that if one increases, the other also increases, and if one decreases, the other decreases. In a direct proportion, one quantity depends directly on the other.

In the unitary method, we find first the value of a unit and then multiply that with the number of units given.

Unitary Method

e.g. **The cost of 6 pens is $42. Find the cost of 13 pens.**

The cost of 6 pens is $42

The cost of 1 pen is $\frac{\$42}{6}$

The cost of 13 pens is $\frac{\$42 \times 13}{6} = \$91$

Note: The unitary method is possible only when two quantities are in direct proportion.

Hence, we can say that the unitary method is a special case of direct proportion.

Consider some examples on the unitary method. If. 2 balls cost $8, find the cost of 3 balls.

Cost of 2 balls = $8

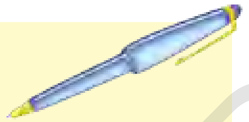
Cost of 1 ball = $8 ÷ 2 = $4

Cost of 3 balls = $4 x 3 = $12

Exercises

① 12 toys cost $72. Find the cost of 4 toys? 72/12 * 4 = $24

② If 6 pens cost $96, how much would each pen cost?
And how much would 18 pens cost? 96/6 = $16
16 * 18 = 2$88



③ If the cost of 8 chocolate bars is 16. Find the cost of 17 chocolate bars.
16/8 * 17 = 34

5.5 Indirect (Inverse Proportion)

Inverse proportion or indirect proportion: is a relation between two quantities such that one increases in proportion as the other decreases, or the other way round.
For example, if a particular job requires 4 men to complete it in 8 hours, then the same job will require 8 workers to perform it in 4 hours. Here the number of workers and the number of hours are in inverse proportion.

Three men can complete a work in 4 hours, how many men will do the job in 12 hours?
As a direct proportion, this ratio would have been $3 : x = 4 : 12$
but this proportion is an example of inverse proportion, therefore, the ratio should be shown in the following way. $x : 3 = 4 : 12$

Now the product of extremes $= 12\ x$

And the product of means $= 3 \times 4$

$$12\ x = 3 \times 4$$

$$x = \frac{3 \times 4}{12}$$

$$x = 1 \text{ man}$$

Example

Find the 4th proportion of the following ratio $2 : 3 :: 10 : \boxed{}$

The product of extremes = The product of means

$$2 \times \boxed{} = 3 \times 10$$

$$\boxed{} = \frac{3 \times 10}{2} = 15$$

Exercises

① Find the 3rd proportion of the following ratio: $6 : 8 :: \boxed{18} : 24$

② Find the 4th proportion of the following ratio: $6 : 9 :: 30 : \boxed{45}$

5.6 Percentage

Percentage is used in many fields such as, business, sports and others. It is a way to show how each quantity is related to another. Percentage means hundredths, we use the symbol % which is read as percent. In a common fraction, like $\frac{2}{3}$, we have a numerator and any number as a denominator.

A percentage is a special fraction with only one denominator (100).

Hence, 50% 50 percent) means $\frac{50}{100}$ or $\frac{1}{2}$. Conversely, a fraction can be changed to a percentage as follows: $\frac{3}{4}$. This is a common fraction with a denominator of 4. In percentage, the denominator must be 100. So, the change of $\frac{3}{4}$ to percentage will require changing the denominator to 100.

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

Rules:

To express a percentage as a fraction divide it by 100.

e.g.

1. Express 12 percent as a fraction.

$$12 = \frac{12}{100} = \frac{3}{25}$$

2. Express $12\frac{1}{2}$ percent as a fraction.

$$12\frac{1}{2} = \frac{25}{2} = \frac{25}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{1}{8}$$

3. Express 25 percent as a decimal fraction.

$$25 = \frac{25}{100} = 0.25$$

Note: **When a number is divided by 100 the decimal point is shifted towards the left by two places.**

4. Express $2\frac{1}{2}$ percent as a decimal fraction.

$$2\frac{1}{2} = \frac{5}{2} = 2.5 = \frac{2.5}{100} = 0.025$$

Note: **When there is only one number before the decimal point, then a zero is added on the left and the decimal point is placed before the zero.**

Note: To express a fraction as a percentage, multiply it by 100.

5. Express $\frac{3}{4}$ as a percentage.

$$\frac{3}{4} = \frac{3}{4} \times 100 = \frac{300}{4} = 75\%$$

6. Express 0.06 as a percentage.

$$0.06 = 0.06 \times 100 = 6\%$$

Note: **When a number is multiplied by 100, the decimal point is moved two places to the right.**

Note: To find the percentage of a given quantity, multiply the quantity by a fraction whose numerator is the given rate and whose denominator is 100.

7. Find 40 percent of \$300.

$$40\% \text{ of } \$300 = \frac{40}{100} \times 300 = \$120$$

8. 25 percent of a number is 50. Find the number.

$$25\% \text{ of } x = 50$$
$$\frac{25}{100}\, x = 50$$
$$x = \frac{5 \times 100}{25}$$
$$x = 200$$

Thus, the number is 200.

Exercises

① Change the following into percentages:

ⓐ $\frac{1}{2}$ 50%

ⓑ $\frac{2}{5}$ 40%

ⓒ 1 100%

ⓓ 0.35 35%

ⓔ 0.05 5%

ⓕ 2.5 250%

ⓖ 0.12 12%

② Change the following percentages into common fractions:

ⓐ 40% 2/5

ⓑ 55% 11/20

ⓒ $5\frac{1}{2}$% 11/200

ⓓ $10\frac{1}{2}$% 21/200

③ Change the following percentages into decimal fractions:

ⓐ $3\frac{1}{2}$% 0.035

ⓑ 6% 0.06

ⓒ 8.8% 0.088

ⓓ 10.03% 0.1003

④ Find the values of the following:

ⓐ 15% of \$2,000
15/100 * 2000 = \$300

ⓑ $2\frac{1}{2}$% of \$4,000
0.025 * 4000 = \$100

ⓒ 25.5% of \$200
25.5/100 * 200 = \$51

⑤ Sixteen percent of a number is 64. Find the number. 16/100 * z = 621
z= 64 * 100/16 = 400

⑥ A boy spends \$300 which is equal to 75% of his pocket money.
How much is his pocket money? 75/100 * y = \$300
y = \$400

⑦ A town has a population of 1,800 men. If 30 percent of the men work as farmers, how many farmers are in this town? 1800 * 30/100 = 540

5.7 Commission

A fee paid for services, usually a percentage of the total cost.

Example

Lee's gallery sold Peter's painting for \$500, so Peter paid them 10% commission (\$50).

Find the commission on a sale of goods for \$2,500 at 5%.

$$\text{Commission} = 5\% \text{ of } \$2{,}500$$
$$= \frac{5}{100} \times 2{,}500$$
$$= 75$$
$$\text{Thus, commission} = \$75$$

Exercises

① Leen makes money by commission rates, she gets 10% of everything she sells, if she sold 25000 this month, what is her salary for the month?

25000 * 10/100 = 2500

② Sue earns a 5% commission on each mobile she sells, if each mobile costs \$200, and she sells 31 mobiles to the mobile store, how much commission will Sue earn?

5/100 * 200 = 10
10 * 31 = 310

5.8 Discount

A discount means a reduction in price. It is the percentage that is subtracted from a number. For example, a 10% discount of $300 is $30. So, if an item that sells for $300 is offered at a 10% discount, it can be bought for $270.

On a clearance sale, the price of a computer was reduced from $500 to $450. Find the discount percent.

Original price = $500
Reduced price = $450
Discount = original price – reduced price
= 500 - 450
= $50

$$\text{Discount \%} = \frac{\text{Discount}}{\text{Original Price}} \times 100$$
$$= \frac{50}{500} \times 100$$
$$= 10$$

Thus, discount = 10%

Example

The written price of an article is $250. A discount of 10% is given. Find the discount and the reduced price of the article.

Original price = $250
Discount = 10% of $250

$$= \frac{10}{100} \times 250$$
$$= 25$$

Reduced price = Original price – discount
= 250 - 25
= 225

Thus, discount = $25
and reduced price = $225

Exercises

1. Find the sale price of an item that has a list price of $24 and a discount rate of 50%. 24 * 0.5 = $ 12

2. Test whether the following proportions are true or false.

ⓐ 4 : 6 :: 3 : 2 F
ⓑ 8 : 6 :: 4 : 3 T
ⓒ $\frac{1}{4} : \frac{1}{5} :: 5 : 4$ F
ⓓ $\frac{2}{4} : \frac{3}{5} :: 5 : 6$ T

3. Fill in the ☐

ⓐ $\boxed{4}$: 6 :: 2 : 3
ⓑ 9 : 27 :: 3 : $\boxed{9}$

4. If 5 apples cost $30, what will be the cost of 9 apples? 30/5 * 9 = $54

5. A tree grows in 5 months by 62 cm. What will be its growth after 9 months? 5/62 * 9/X = 111.6

6. Two dozens of pens cost $72. Find the cost of 50 pens. SKIP

7. A car covers a distance of 180 km in 3 hours. What distance will it cover in 7 hours travelling at the same speed? 3/180 = 7/x
= 420 km

8. The weekly expenditure of a family is $840. Find their expenditure for a month (30 days). 840 * 30 / 7 = $ 3600

9. The wages of a manger are $150 per day. Find his income for (i)......... (ii) .... skip

10. If 39 men earn $2340 in a day's work, what would 91 men earn in a day? 2340 * 91 / 39 = $ 5460

11. If (25) computers cost $12,500. Find the cost of (35) computers. 12500 * 35/25 = 17500

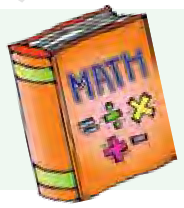
5.9 Problem Solving

① A man saves 35% of his salary.
What percentage does he spend? 65%

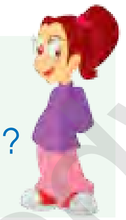
② There is a discount of 25% on school bags. A bag price is $350.
What will be the discounted price? $25/100 * 350 = 87.5$

③ A bookseller receives $2,400 as commission from a publisher.
Find the amount of the sale, if the rate of the commission is 12 percent. $2400 * 100/12 = \$ 20000$

④ A sales manager in a factory gets $2,000 as salary and a 5 percent commission on the goods sold. He sells goods worth $12,500 in a month. Find his income for the month. $5/100 * 12500 = 625 + 2000 = \$2625$



⑤ A mathematics book is 3.5 cm thick, and a science book is 4.5 cm. What is the ratio of the mathematics book thickness to that of the science book? 7/9

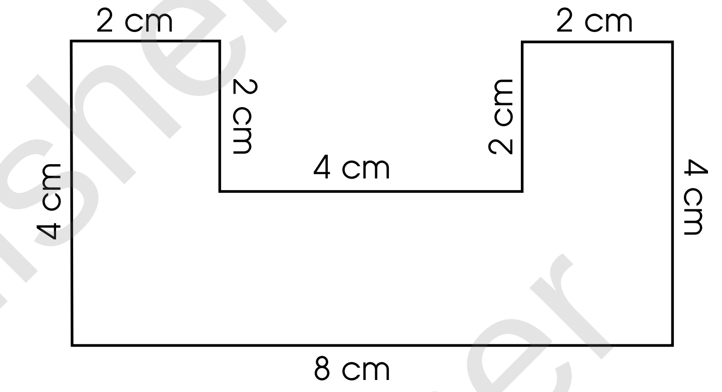


⑥ A girl got 60 percent of the objectives correct by doing 18 objectives correctly. What was the total number of objectives?
$60/100 * y = 18$
$y = 18 * 100 / 60 = 30$

⑦ By covering 20 km a man has covered 25 percent of his journey.
What is the total length of his journey? $y = 20 * 100/25 = 80$

6.1 Perimeter

Perimeter is the distance around the boundary of a plane closed figure, in other words, the perimeter is the sum of the length of the sides of a shape.

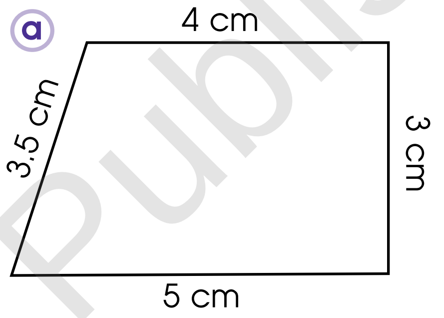


The perimeter here is
4 + 2 + 2+ 4 + 2 + 2 + 4 + 8 =
28 cm

Exercises

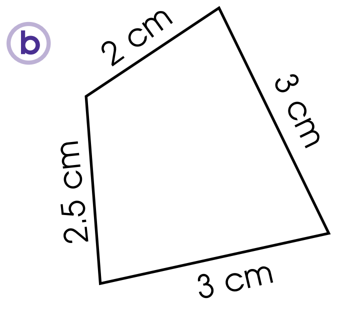
① Find the perimeter in the following:

ⓐ



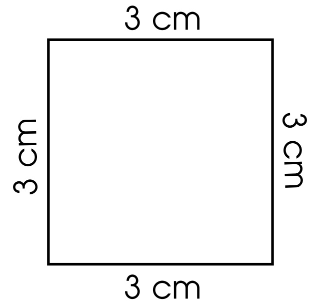
P = 4 + 3.5 + 3 + 5
= 15.5

ⓑ



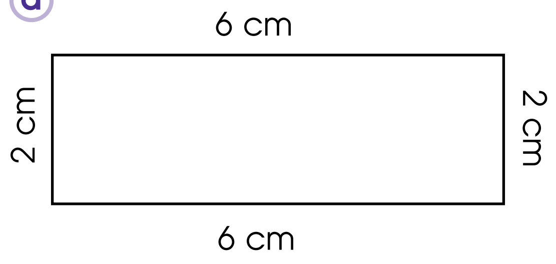
P = 10.5

ⓒ



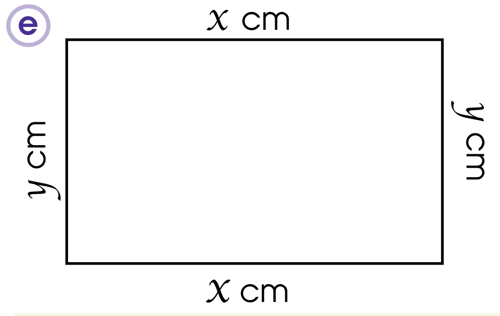
P = 12

ⓓ



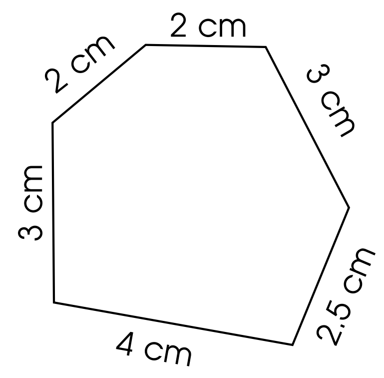
P = 16

ⓔ



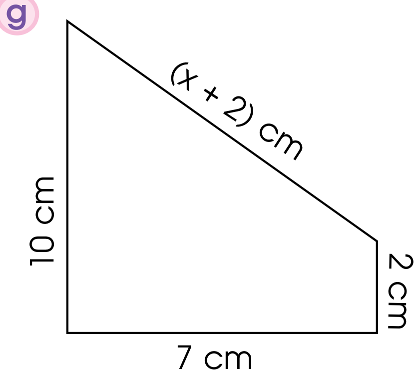
P = 2x + 2y

ⓕ



P = 16.5

ⓖ



P = x + 21

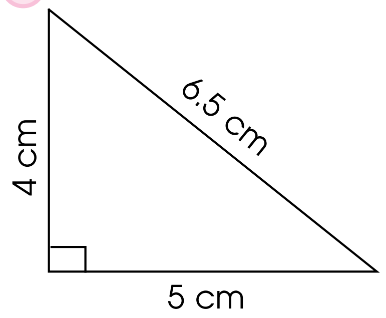
Look at (e) and (d). What do you notice?

What is this shape called? ....

Can you calculate the perimeter in another way? ....

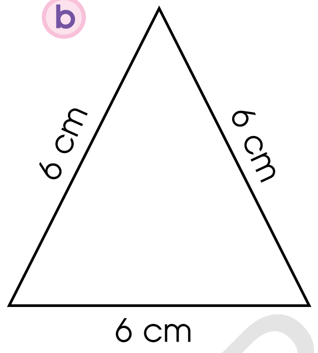
② Find the perimeter in the following triangles.

ⓐ



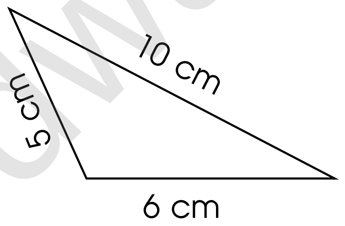
P = 4 + 6.5 + 5
= 15.5 cm

ⓑ



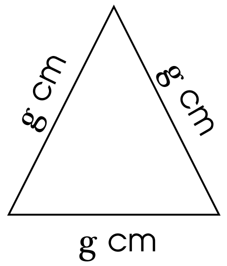
P = 18 cm

ⓒ



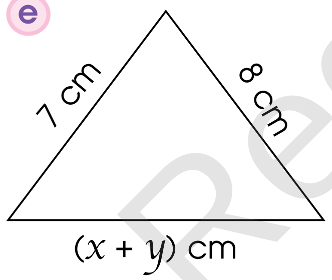
P = 21 cm

ⓓ



P = 3 * g

ⓔ



P = 15 + x + y

Look at (b) and (d). What do you notice?

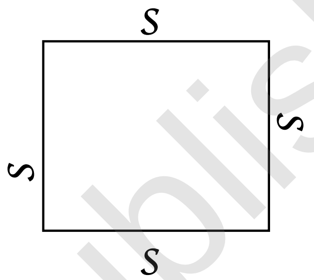
What is the type of this triangle?

Perimeter of Regular Shapes

① **The perimeter of a square can be found by 4 x S, S: the length of side.**

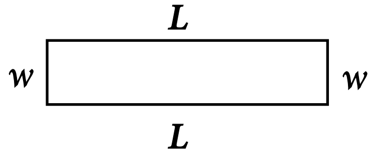
② **The perimeter of a rectangle can be found by $2L + 2w$, L = length. w = width.**

③ **The perimeter of an equilateral triangle can be found by 3 x n, n = the length of side.**



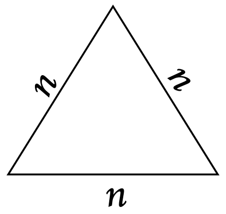
P = $S + S + S + S$

P = $4(S)$



P = $L + w + L + w$

P = $2L + 2w$

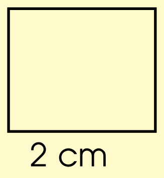


P = $n + n + n$

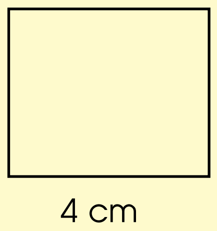
P = $3(n)$

Exercises

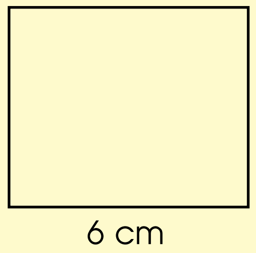
① Find the perimeter of the following **squares**.



8

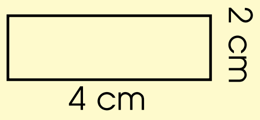


16

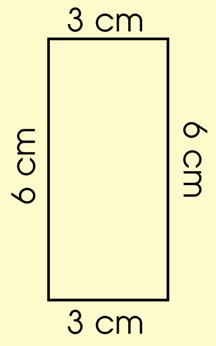


24

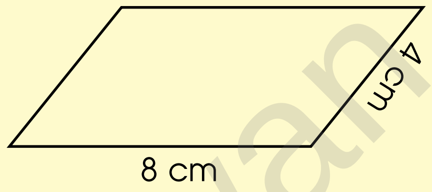
② Find the perimeter of the following **rectangles**.



12

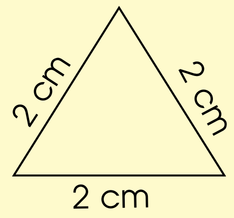


18

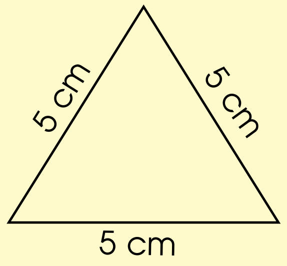


24

③ Find the perimeter of the following **triangles**.



6



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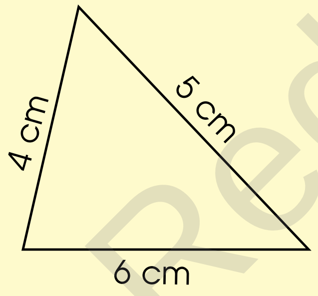


15

④ If the perimeter of a square is 81 cm, what is length of each side? (Check your answer). $P = 45$

$81/4 = 20.25$ cm

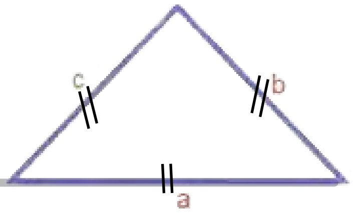
⑤ If the perimeter of a rectangle is 24 cm, and the length = 9 cm, what is its width? (Check your answer). $24 = 2 * 9 + 2 * w$

$w = 3$ cm

⑥ If the perimeter of a rectangle is 80 cm, and the width = 8 cm what is its length? (Check your answer). $80 = 2 * 8 + 2L$

$= 32$ cm

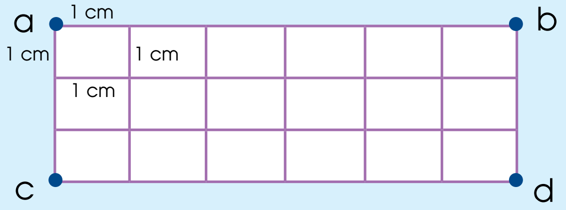
⑦ If the perimeter of an equilateral triangle is 49 cm, what is the length of each side? (Check your answer). $49/3 - 3n /3 = 16.33$ cm



6.2 Area

Definition: Area is the amount of space inside a figure. We use square units to describe area.

1. What is the name of this figure abcd?____________
2. How many rows of squares are there? ____________
3. How many squares are there in each row? ____________
4. How many squares are there in the rectangle? ____________
5. What is the area of abcd? ____________



Note: **The square units on each side of the square = 1 cm.**

To find the area of the rectangle abcd, just find the sum of the squares inside. You will get area = 18 squares, but if we want to know the area of the the rectangle abcd as a unit:

the length = 6 cm (6 squares).

and the width = 3 cm (3 squares).

Is there any relation between the length, and the width that resulted in calculating the area as 18 square units?

Find the area of the rectangle EDFG given the shown measurements.

The area = 8 x 2 = 16 cm^2

The area of a square = $S \times S = S^2$

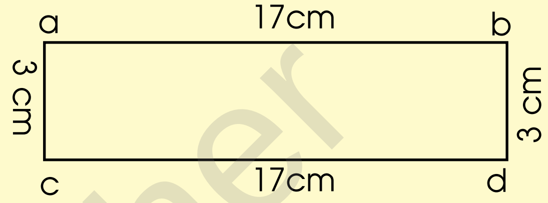
S **= length of side.**

The area of a rectangle = $L \times W$, L**: length.**

W**: width.**

Exercises

1. Find the area of a square with a length of side = 3 cm. $3^2 = 9$ cm^2
2. What is the length of side in a square that has the area of = 100 m^2.
   $\sqrt[2]{100} = 10$ cm
3. Find the area of the rectangle abcd given the shown measurements:

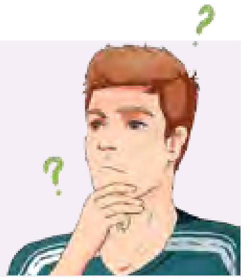


$3 * 17 = 51$ cm^2

4. Rectangle vwxy has an area of 100 m2.
   vw = 22 m
   What is the length of width of the rectangle vwxy?
   100/22 = 22w/22 = 4.54 m

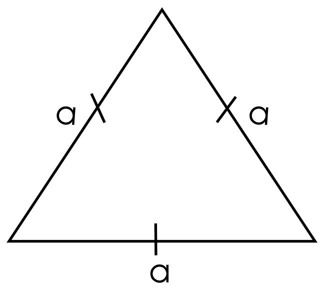
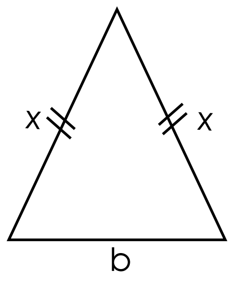
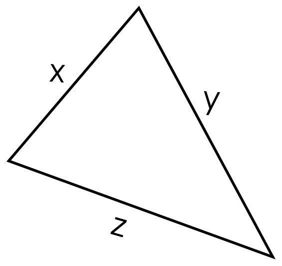
Think, discuss, analyze, and give examples.

- Shapes with the same areas may have different perimeters.
- Shapes with the same perimeters may have different areas.

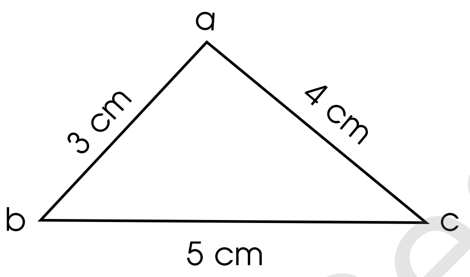


6.3 Perimeter and Area of Triangles

The perimeter of a triangle equals the sum of the lengths of its three sides.

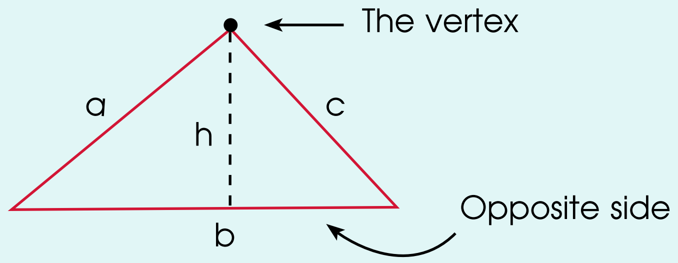
Equilateral Triangle	Isosceles Triangle	Scalene Triangle
P = a+a+a = 3a	P = 2(x) + b	P = x + y + z
		

So, to find the perimeter of the triangle abc bellow:



The area of a triangle = $\frac{1}{2}$ x Base x Height

The height of a triangle is each of the perpendicular lines drawn from one vertex to the opposite side.

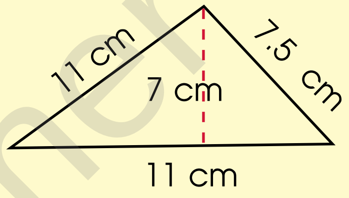


Area = $\frac{1}{2}$ x b x h

Exercises

1- Find the following:

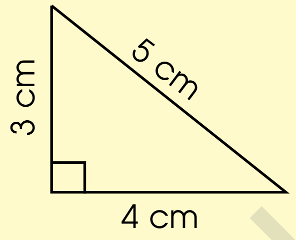
a



Scalene Triangle

- What is the type of this triangle?
- Find the perimeter of the triangle abc. P = 11 + 7.5 + 11 = 29.5
- Find the area of abc. 1/2 * 11 * 7 = 38.5

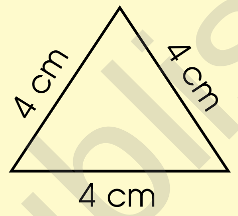
b



Scalene (Right angled)

- What is the type of this triangle?
- Find the perimeter of the triangle. P = 12
- Find the area of the triangle. $1/2 * 3 * 4^2 = 6$

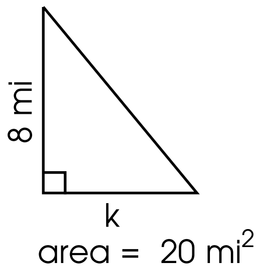
c



- What is the type of this triangle? Equilateral
- Find the perimeter of the triangle. P = 12
- Find the area of the triangle. $\frac{\sqrt{3}}{4} (4)^2 = 4\sqrt{3}$

(Hint): Area of equilateral triangles = $\frac{\sqrt{3}}{4}$ $(\text{side})^2$

2- What is the missing length?



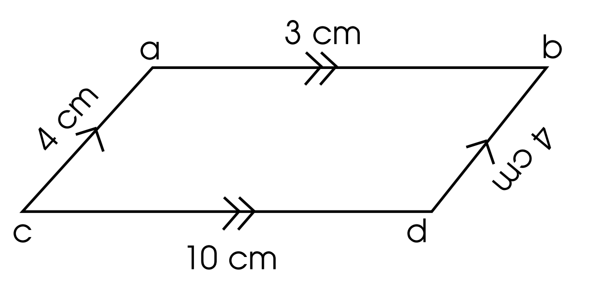
20 = 1/2 * 8 * k
k = 5 mi

3- Find the area of a triangle with a base of 16 feet and a height of 3 feet. 1/2 * 16 * 3 = 24

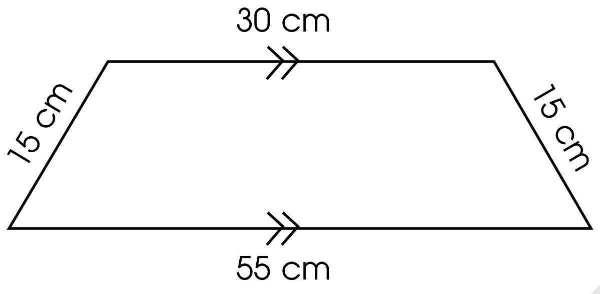
4- Find the height of a triangle that has an area of 36 in^2 and a base of 6 in. 36/3 = 3h/3 = 12 in

6.4 Perimeter & Area of Parallelograms & Trapezoids

We learned that the perimeter is the distance around a two- dimensional closed shape, or it is the length of the boundary.
Find the perimeter of the shapes below:

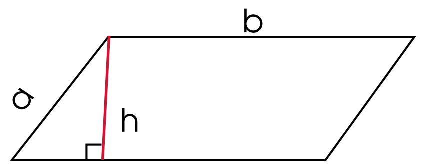
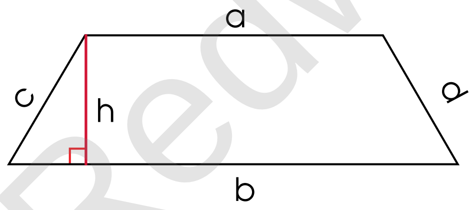


(parallelogram)
Perimeter = ____________

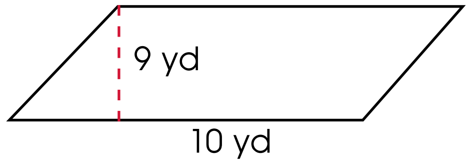
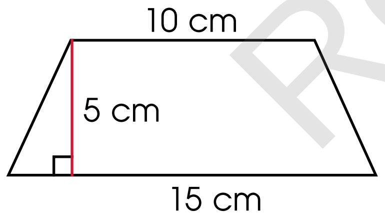


(trapezoids)
Perimeter = ____________

Search for the definition of parallelograms and trapezoids and explain it to your colleagues next lesson.

Area of parallelogram	Area of trapezoids
 Area = b h	 Area = $\frac{a + b}{2}$ h

Exercises

(1) Find area	(2) Find area
	

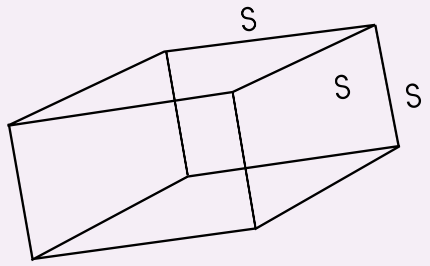
6.5 Volume of Cubes and Rectangular Prisms

A three dimensional (3D) object.

The 3 dimensions are called width, depth and height.

Examples include, spheres, cubes, pyramids and cylinders, which you can google.
A cube is a three dimensional solid object bounded by six square faces or sides.
Every three faces meet at a vertex.

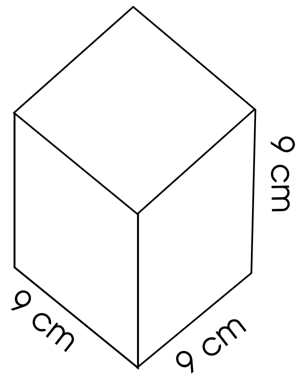
Volume = S x S x S = S^3



Exercises

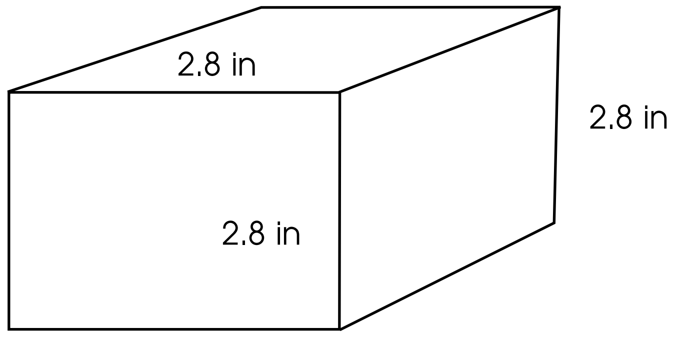
1 What is the volume?

$9^3 = 729$



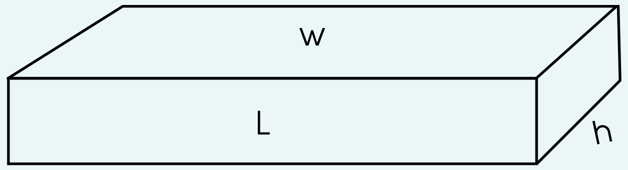
2 What is the volume?

$(2.8)^3 = 21.952$



③ The volume of a cube is 1000 cm. 5 = 10
What is the area of every face? 100 cm^2
What is the perimeter of every face? 40 cm

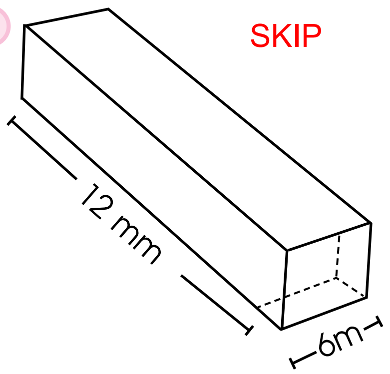
A rectangular prism is a solid (3 dimensional) object which has six faces, each of which has the same cross-section length. This makes a prism also a "cuboid".



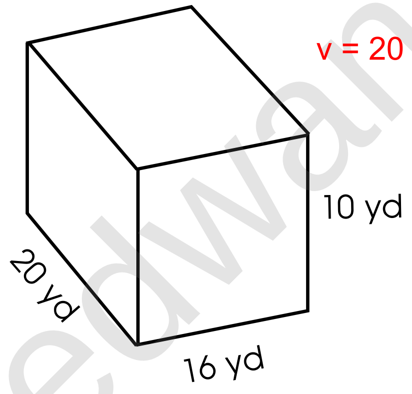
Volume = Lwh

④ Find the volume of these rectangular prisms.

ⓐ SKIP

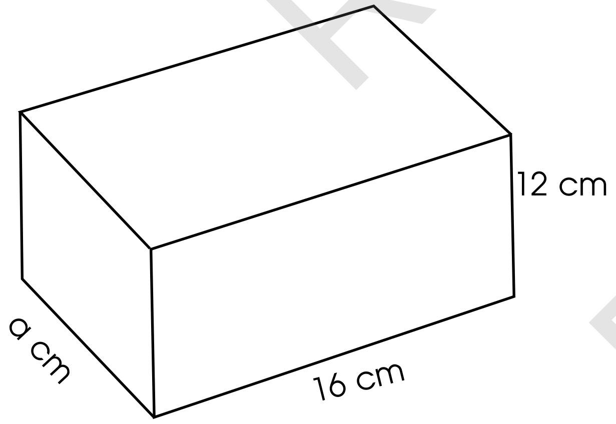


ⓑ v = 20 * 16 * 10 = 3200 ³

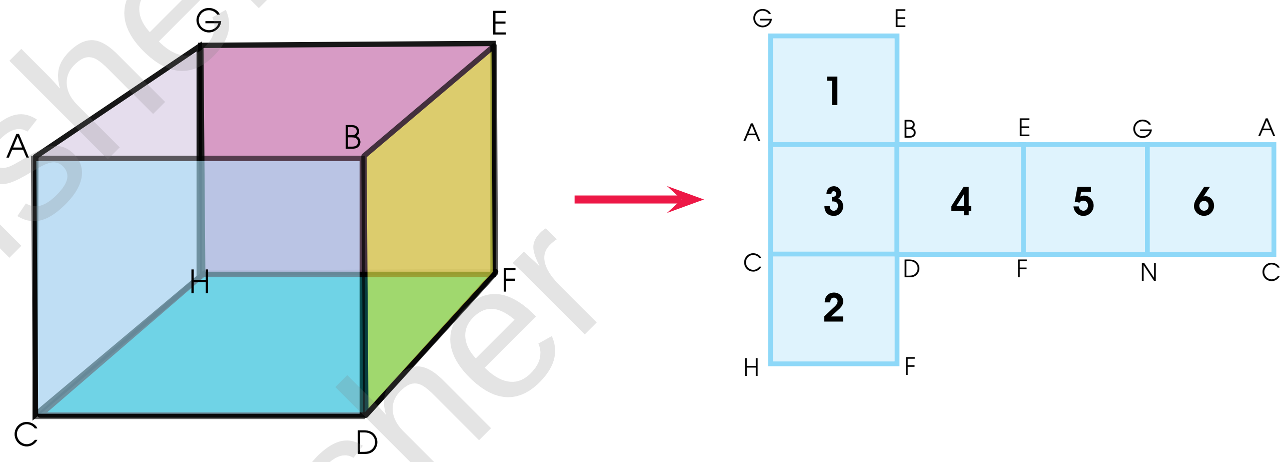


⑤ The volume of this rectangular prism is 2688 cm^3, what is the value of a as shown?

12 * 16 = 192
2688/192 = 14
a = 14



6.6 Surface Area of Cubes and Rectangular Prisms

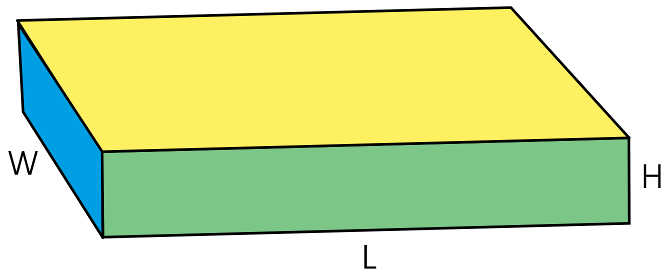


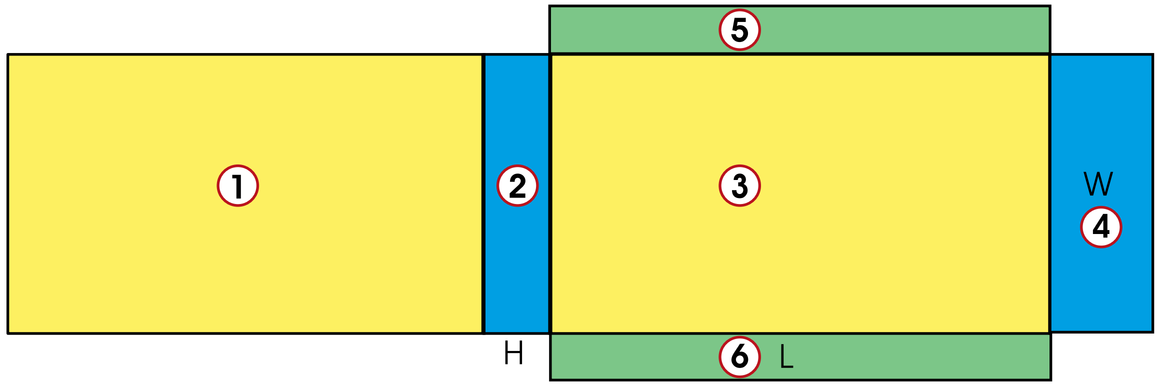
We have a cube with a length of side = y

The Area for each face = y^2 so we have 6 equal sides and 6 equal faces.
Then, the Surface Area = $6y^2$

Surface Area of Rectangular Prisms

Start with a right rectangular prism as shown below. Call the length L, and the height H.





The Surface Area here = ①+②+③+④+⑤+⑥

= WL + WH + WL + WH + HL + HL

= 2WL + 2WH + 2 HL

Exercises

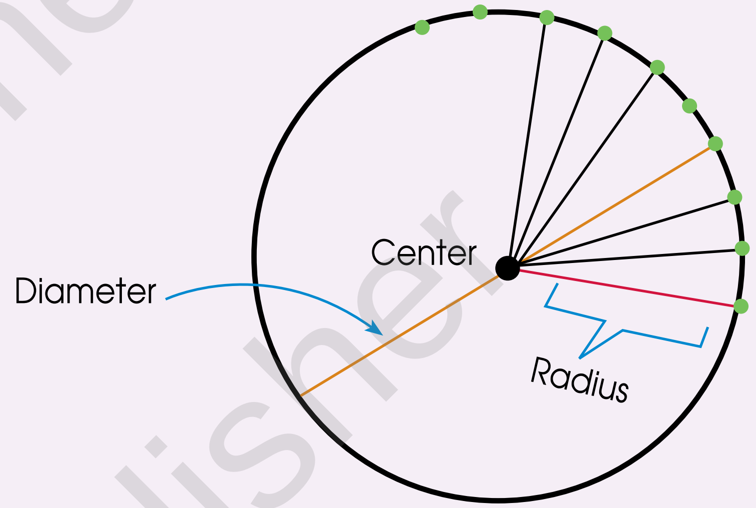
① Find the surface area of a rectangle prism with a length of 6 cm, a width of 5 cm, and a height of 10 cm. (2 * 6) + (2 * 5) + (2 * 10) = 42

② Find the surface area with a length of $\frac{1}{2}$ cm, a width of 8 cm, and a height of $\frac{1}{4}$ cm.

(2 * 1/2) + (2 * 8) + (2*1/4) = 17.5

6.7 Circles (Area & Circumference)

A Circle is: It is the set of all points in a plane that are at an equal distance from a given point (the center).

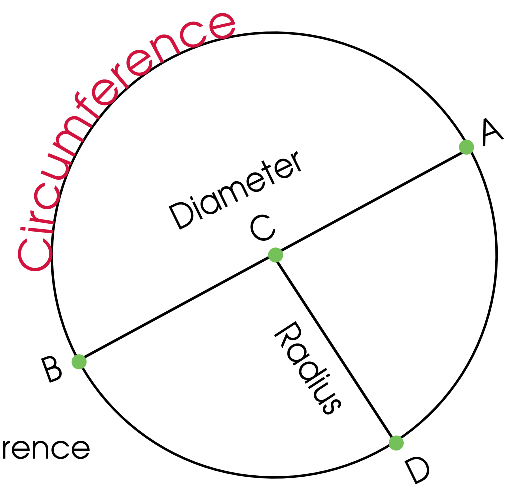


The radius is the distance from the center of the circle to its edge.

The diameter of a circle is a segment whose end points lie on the circle and whose midpoint is the center of the circle and when someone says that something "lies on the circle" that means it's on the outline that traces the circle.

Diameter = AB

Radius = CA, CB, CD



- **How many diameter do we have in a circle?**
- **How many radii can we draw in a circle?**

The distance around a circle is called circumference

$= 2r\pi$ or πr

When r = radius, diameter = 2r.

Examples

① Find the circumference of a circle whose r = 4 cm.

Circumference = $2\pi r = 2(4)\pi = 8\pi$ cm

② Find the circumference of a circle whose r = 3 cm, $\pi = \frac{22}{7}$

Circumference = $2\pi r$

$= 2(3)\left(\frac{22}{7}\right)$

$= \left(\frac{132}{7}\right)$

$= 18.85$ cm

The value of π is $\frac{22}{7}$ or 3.14

$\pi = \frac{\text{Circumference}}{\text{Diameter}}$ or Circumference = π x Diameter

The area of a circle is the number of square units inside that circle $A = \pi r^2$

Exercises

① The radius of a circle is 3 inches, what is the area? $\pi = 3.14$ $\pi r^2 = 28.26$

② The diameter of a circle is 8 cm, what is the area? $\pi r^2 = 50.24$

③ The area a circle is 78.5 square meters, what is the radius? $78.5/3.14 = \sqrt{25} = 5$
$r = 5$

④ The circumference of a circle is 18.28 m, what is the area? $2\pi r = 18.28/2 = 9.14/3.14 = 2.91$
area $= \pi r^2 = 26.58$

⑤ The area of a coin is 3.14 square centimeters, what is the radius and circumference of the circle? $\pi r^2 = 3.14/\pi$ Circumference $= 2\pi r = 6.28$
$r = 1$

7.1 The Basic Counting Principle

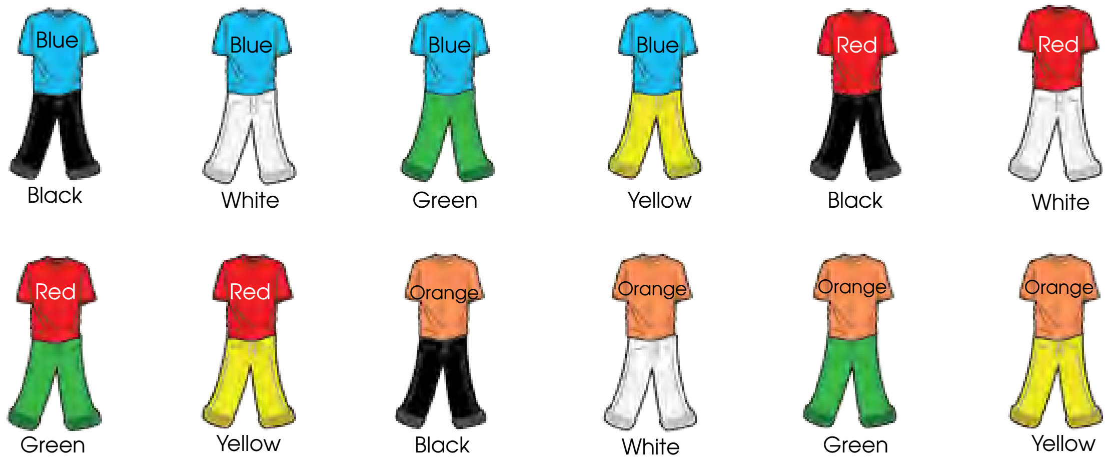
Probability is a branch of mathematics that is concerned with the analysis of random phenomena. The outcome of a random event cannot by determined before it occurs, but it may be any of several possible outcomes. The actual outcome is to be determined by chance.

Statistics is a branch of mathematics dealing with the collection, analysis interpretation, presentation and organization of data like this table.

A	B	C
23	45	30 - 10
70	33	20 - 39
100	45	⋮
⋮	⋮	⋮

If you have 3 shirts: the first is blue, the second is red, and the third is orange, and you have 4 pairs of pants: black, white, green, and yellow. How many outfits do you have? Can you count them?

That means 3 x 4 = 12 different outfits.

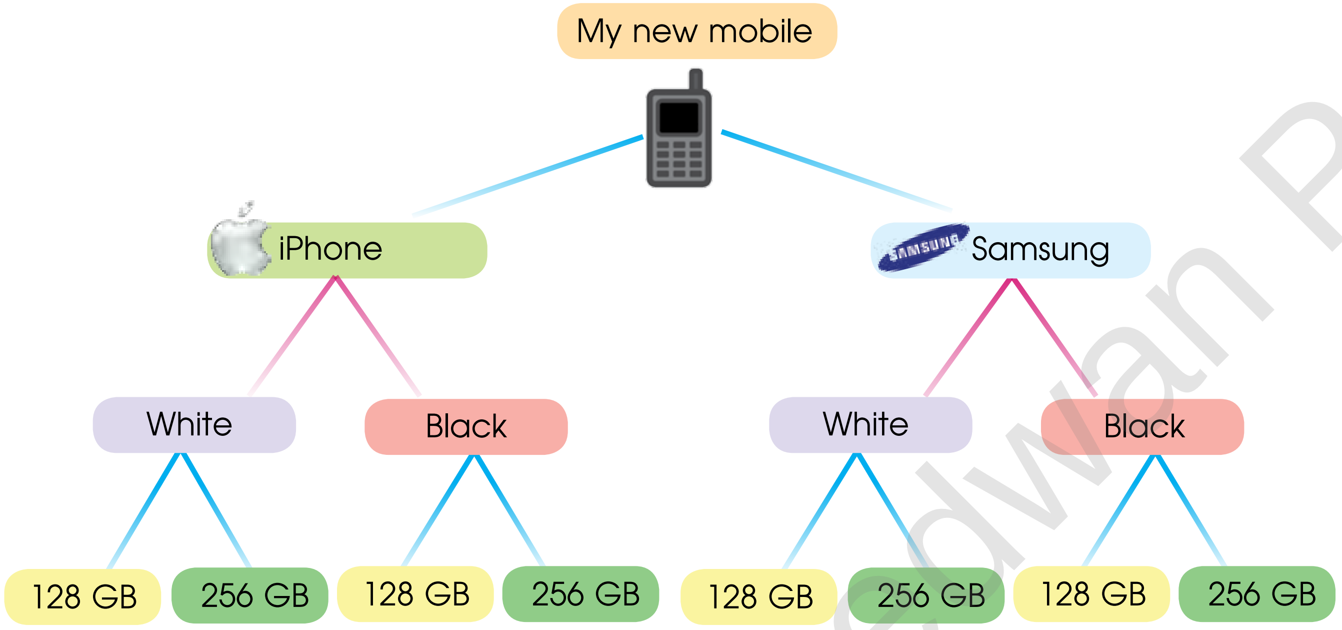


Look at this example:

You are buying a new mobile. There are 2 types available in the store.

Suppose that there are 2 colours available:
White and black.
There are 2 internal memories: 128 GB and 256 GB.
How many total choices do you have?

Look at this tree diagram:



Total choices = 2 x 2 x 2 = 8 choices. Count them now.

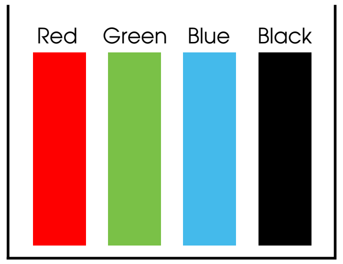
Exercise

Lee is planning an activity with his wife. He plans to watch a movie, go out for dinner, and then attend a sporting event. He is deciding between 5 movies, 8 restaurants, and 2 sporting events.

How many different activities can Lee plan? 5 * 8 * 2 = 80

7.2 Probability Experiments

Probability is the chance that an event or situation will happen.
Below, there's a box that contains 4 differently-colored cards.



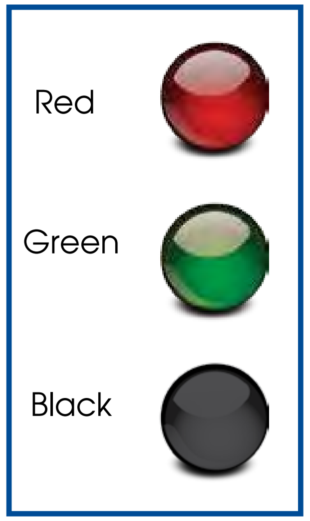
So, close your eyes and pick a card.
What color do you think it is?

The outcome will be one of them:
Red, green, blue or black.

Each outcome has 1 out of 4 chances of happening, so each outcome is equally likely.

- The probability to get a green card = $\frac{1}{4}$ ← Number of green outcomes. ← Total possible outcomes.
- The probability to get a red card = $\frac{1}{4}$
- The probability to get a blue card = $\frac{1}{4}$
- The probability to get a black card = $\frac{1}{4}$
- The probability to get an orange card = $\frac{0}{4} = 0$ ← Number of orange outcomes.

Look at this box:



① The probability to get a red ball = ____________

② The probability to get a green ball = ____________

③ The probability to get a black ball is = ____________

Exercises

① You roll a 6 - sided dice, the outcomes are {1, 2, 3, 4, 5, 6}.

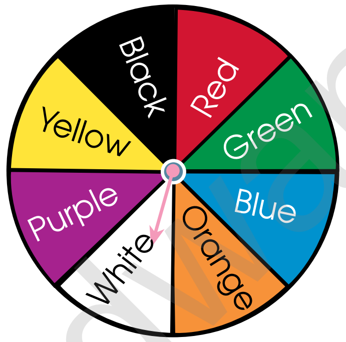
1- What is the probability of (5) to show up? P (5) = 1/6

2- What is p (6)? 1/6

3- What is p (10)? 0

4- What is p (0)? 0

② You spin the spinner once. Write the outcomes as {black, ..., ...}



1- What is the p (red)? 1/8

2- What is p (purple)? 1/8

3- What is p (white)? 1/8

4- What is p (pink)? 0

5- What is p (green) + p (white) + p (yellow)? 3/8

6- What is p (green) + p (white) + p (purple) + p (red) + p (blue) + (black) + p (purle) + p (orange)? 8/8 = 1

③ What do you think about the probability that p (x) = 2? Is it a possible answer? Can I get an answer of probability that's more than (1)? No we can't

$$\overline{X} = \frac{\sum X}{N}$$

7.3 Mean (Average)

Definition: The mean of a set of data is their average.

The idea of 'average' or arithmetic meaning is familiar to us. We always read statements such as: "The average score made by a player in a series of matches......" The arithmetic mean of a set of numbers is the sum of scores divided by the number of matches. Now consider the following example:

e.g. A salesman sold 84 packets of biscuits in a week. The number of packets sold each day of the week is given in the table below. Find the average of sold packets.

Day	Sat	Sun	Mon	Tue	Wed	Thu	Fri
No. of packets	1	18	16	14	10	20	5

(arithmetic mean) $= \frac{\text{Sum of no. of packets}}{\text{No. of days}}$

$= \frac{1 + 18 + 16 + 14 + 10 + 20 + 5}{7}$

mean $= 12$

So, the average of sold packets is 12 packets per day.

To find the mean:

① Find the sum of the data.

② Divide the sum of items by the set of data.

Exercise

Find the average of the following by sets of numbers.

ⓐ 4, 6, 7, 8, 2, 5,10 42/7 = 6

ⓑ $1\frac{2}{5}$, $3\frac{1}{5}$, $5\frac{2}{5}$ 9*1 /3 = 3

ⓒ 10.8, 11.5, 10.9, 12.5, 11.8, 10.3

67.8 / 6 = 11.3

ⓓ 138, 164, 150, 148, 152, 144, 168

1064/7 = 152

Weighted Average

An average resulting from the multiplication of each component by a factor reflecting importance. In calculating an average involving several quantities, it is often seen that the value of one affects the answer more than the value of the others. This type of average is called 'weighted average'.

Examples

1 Three pencils were bought for $2 each and 9 pencils were bought for $6 each. What is the average price per pencil?

Price of 3 pencils $2 each $= (2 \times 3) = 6$

Price of 9 pencils $6 each $= (9 \times 6) = 54$

Price of (3+ 9) 12 pencils $= 6 + 54$

$= 60$

Average price per pencil $= \frac{\text{Total price}}{\text{Number of pencils}}$

$= \frac{60}{12}$

Average price per pencil = $ 5

This is not the average of $2 and $6. Since fewer pencils were bought for $2 each than for $6 each, the first price has less effect on the average than the second. This is an example of weighted average because it is necessary to attach more importance or weight to one price than the other.

2 A motorist drives 3 hours at 50 km/hr and 2 hours at 45 km/hr. Find his average speed for the whole journey.

Distance covered in 3 hours at 50 km/hr $= 50 \times 3 = 150$ km

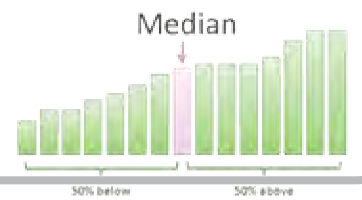
Distance covered in 2 hours at 45 km/hr $= 45 \times 2 = 90$ km

Total distance covered in (3+2) 5 hours = 240 km

Average Speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

$= \frac{240}{5}$

Average Speed = 40 km/hr



7.4 Median and Mode

Median

Is the middle value between the minimum and the maximum value of data arranged regularly.
Then the median of a set of data is the middle number when the data are listed in order form least to greatest.

Examples

1 Find the median for 2.4.6.
The median is '4'.

- The median for an odd number of values is the middle number when the values are arranged in ascending order.
- The median for an even number of values is the mean of the two middle values when the values are arranged in ascending order.

2 Seven students from a class participated in a quiz. Their marks were 20, 84, 95, 60, 91, 65, and 50. Find the median.
We arrange the marks in ascending order of 20, 50, 60, 65, 84, 91, 95. Since we have an odd number of marks, the median is the middle score (65).

3 Suppose, 3 more students from the same class participated in the same quiz, as in the above example, and their marks were 70, 75, and 92. Find the median.
We arrange marks in ascending order: 20, 50, 60, 65, 70, 75, 84, 91, 92, 95. Since we have an even number of marks, the median is the mean of the two middle marks. The two middle marks are 70 and 75.

The median $= \frac{70 + 75}{2}$

The median $= 72.5$

Exercises

① Find the median for every set of data.

ⓐ 3, 2, 0, 4, 9 0, 2, 3, 4, 9 → 3

ⓑ 11, 8, 10, 9, 2, 3, 1, 4 1, 2, 3, 4, 8, 9, 10, 11→4+8/2 = 6

ⓒ 90, 100, 89, 70, 85, 101 70, 85, 89, 90, 100, 101 → 90 + 89/2 = 89.5

② During the first marking period, Sara's math cut scores were 90, 91, 92, 88, 97, 87.

Find the median cut score, then find the average.

87, 88, 90, 91, 92, 97
median = 90 +91/2 = 90.5
avg = 90.83

Mode

The most frequently occurring number in a data set is called 'mode'. It is often useful to know the number which occurs most frequently in a set of numbers.

e.g. The marks obtained by a student in a series of tests are given as follows: 20, 18, 15, 18, 16, 18, 17. The modal mark or the mode of the marks is 18 since it appeared most often.

Exercises

① Find the average, median and mode of the following numbers.

ⓐ 10, 11, 13, 11, 15, 16
Mode = 11
Avg = 12.66
Median = 12

ⓐ 11, 9, 10, 7, 6, 5, 8, 7
Mode = 7
Avg = 7.87
Median = 7.5

② The scores of a basketball team in a series of matches are: 63, 63, 75, 67, 68, 52, 50, 63, 56, 52. 50, 52, 52, 56, 63, 63, 63, 67, 68, 75

ⓐ Find the average score of the matches. 60.9

ⓑ Determine the median score. 63

ⓒ What is the modal score of the matches? 63

③ The following is the number of problems that teacher Lee assigned for homework in 10 different days. Find the mode of the following numbers: 8, 11, 9, 14, 13, 18, 6, 9, 10, 13, 14, 9, 6, 12, 13, 16, 14 mode = 9, 13, 14

Note : $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$, $\text{Distance} = \text{Speed} \times \text{Time}$

① The average age of 16 boys is 15 years and of 24 girls is10 years. Find the average age of all. $\frac{15 + 10}{2} = 12.5$

② 30 kg of coal costing $8 per kg is mixed with 10 kg of coal costing $6 per kg. Find the average per kg. $7

③ The average score of 8 matches in one day was 230, and that of 12 matches was 200, find the average of all. 215


F = G m1 m2 / d2
E =
+ 1/16 + 1/25 + … = π2/6
F(n) = ((φ)n − (−1/φ)…)/√5
∂2V/∂S2 + rS ∂V/∂S + ∂V/∂t − rV = 0
L(v2)
1/c ∂E/∂t
Σn 1/ns
(x−μ)2/2σ2
∫−∞∞
iπ