NEW
FAVOURITE
MATH

New Favourite Math Series (Six levels) is designed in accordance with the Principles and Standards for School Mathematics (PSSM) that focus on mathematics content and its methods of teaching, and are produced by the National Council for Teachers of Mathematics (NCTM).

New Favourite Math carefully observes:

- using mathematical concepts, generalizations and laws in a smooth way.
- employing points of stimulating all forms of mathematical thinking.
- age, in addition to the developmental and physiological features of the students.
- employing modern constructivist teaching methods.
- using realistic pictures and portrayals far away from fantasy.
- connecting mathematics with life to highlight the importance of mathematics through mathematical problems.
- designing teaching resources and aids.
- the modernity of the content and keeping up with technological advancement.
- creating the spirit of challenge and competition.
- steering clear of routine and unnecessary repetition.

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FOR PUBLISHING & DISTRIBUTION

NEW FAVOURITE MATH

Level E

www.redwanpublisher.com

Moern Book Center

ISBN 9789661512770

We would like to thank the editors and our designers, who all contributed to the development of New Favourite Math.

We would like to dedicate this course to the teachers around the world who will bring New Favourite Math to life in their classrooms.

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The publisher would also like to thank Prof. Brian Marshall and Dr. Mark Erans for their efforts in “New Favourite Math” Project.

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Second Edition 2018

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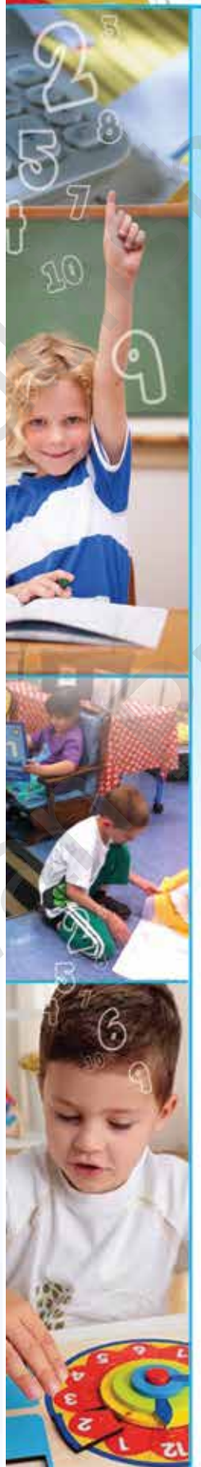

New Math 5th Grade

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1.1 Place Value and Number Names

Numerals	Numbers in words
5	Five
15	Fifteen
55	Fifty-five
237	Two hundred and thirty-seven
1826	One thousand, eighty hundred and twenty-six
14536	Fourteen thousand, five hundred and thirty-six
275508	Two hundred and seventy-five thousand, five hundred and eight
3000000	Three millions

Exercises

1- Write these words in numerals.

a) Fourteen thousand, three hundred and nine: 14309

b) Thirty three thousand, one hundred and ten: 33110

c) Two million, seven thousand, and twenty-two: 2000 7022

2- Express these numerals in words.

a) 732555: seven hundred thirty-two thousand five hundred fifty-five

b) 2100321: two million one hundred thousand three hundred twenty-one

c) 75169: seventy-five thousand one hundred sixty-nine

Expanded Notation

Writing a number to show the value of each digit.

It is shown as a sum of each digit multiplied by its matching place value (units, tens, hundreds, etc.)

5732 = 5 thousands + 7 hundreds + 3 tens + 2 ones

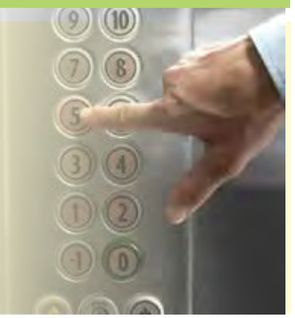
5000 + 700 + 30 + 2

3- Write the following standard notation numbers in expanded form.

a) 32701 — 32 thousands + 7 hundreds + 1 — 32000 + 700 + 1

b) 8035 — 8 thousands + 3 tens + 5 ones — 8000 + 30 + 5

1.2 Positive and Negative Numbers



Look at these examples.

1- Temperature below zero degree.

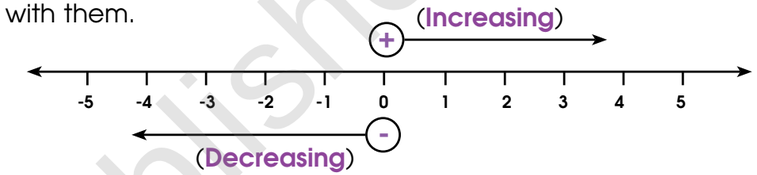
2- The basement floor in a building.

3- When you borrow money from your friend.

4- Elevations below sea level.

We noticed that there are numbers less than zero (below zero).

Look at the number line. We call them negative numbers and use the negative sign (-) with them.

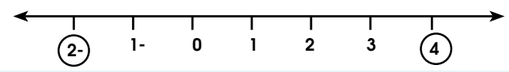


Examples

Use > or < in the ○ and show which one on the right on the number line:

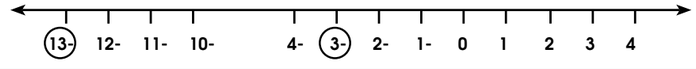
a) (-2) < 4: 4 is greater than (-2)

4 is on the right side of (-2) on the number line.



b) (-12) < (-3) is greater than (-12).

(-3) is on the right side of (-13) on the number line.



Exercises

Use > or < in ○ with reasoning:

a) 314 > (-973).

b) (-1) < $(\frac{-1}{2})$.

c) 0.01 > (-72).

d) (0) > (-82).

1.3 Ordering Numbers

Ascending order: the ordering from smallest (first) to greatest (last)

Example

32751 , 345167 , 430012

smallest greatest

Remember going upstairs

Descending order: the ordering from greatest (first) to smallest (last)

Example

430012 , 345167 , 32751

greatest smallest

Remember going downstairs

Exercises

1- Arrange the digits 0, 2, 4, 6, 8 to make the greatest number you can, then make the smallest one. 86420 / 20468

2- Arrange the numbers in ascending order.

a) 73215, 89321, (-77510) , 100100.

(-77510), 73215, 89321, 100100

b) (-3753), (-75091), (-550), 98.

(-75091), (-3753), (-550), 98

3- Arrange the numbers in ascending order.

a) 632101, 63211, 750310 , (-321).

(-321), 63211, 750310, 632101

b) (-7732), (-982), 98, (-10000).

(-10000), (-7732), (-982) , 98

1.4 Rounded off and Estimated Value

Which of the numbers in the examples below are rounded off?

Wind speed around 100 k/hr.

30000 people are expected to participate.

Today's temperature is 36.4 c°.

I am 132 cm tall.

Exercises

1- Is the number **79842** closer to **79000** or to **79500**? Why?

79000 because 8 is greater than 5

2- Round off these numbers to the nearest hundred.

a) 248 — 200

b) 767 — 800

c) 4871 — 4900

3- Round off these numbers to the nearest thousand.

a) 5291 — 5000

b) 17820 — 18000

c) 26684 — 27000

1.5 Powers and Roots

The power of a number means how many times the number is used in a multiplication.
It is written as a small number to the right and above the base number.

base 4^{2} power

$4^{2} = 4 \times 4 = 16$

In words 4^{2} could be "4 to the power 2"

or simply

4 squared

So, when the power = 2 we use the word "squared".

Note: Another name of power is exponent.

Exercises

1- Complete the following:

Then $5^{2} = 5 \times 5 = 25$. In words: Five squared.

a) $6^{2} = 6 \times 6 = 36$, in words: Six squared.

b) $7^{2} = 7 \times 7 = 49$

c) $8^{2} = 8 \times 8 = 64$

d) $9^{2} = 9 \times 9 = 81$

e) $10^{2} = 10 \times 10 = 100$

f) $11^{2} = 11 \times 11 = 121$

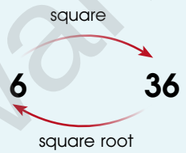
g) $12^{2} = 12 \times 12 = 144$

2- Now the squares are also on the multiplication table:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The origin of 49 = 7 x 7, 100 = 10 x10

Square root going in the opposite direction of powers →



A square root of a number is the value that can be multiplied by itself to give the original number. Then the square root of 36 is 6 because when we multiply 6 by itself, we get 36.

This is the symbol that means "Square root"

If $\sqrt{25} = 5$

$\sqrt{100} = 10$

$\sqrt{16} = 4$

Then, find

a) $\sqrt{49}$ = 7

b) $\sqrt{1}$ = 1

c) $\sqrt{144}$ = 12

d) $\sqrt{81}$ = 9

A number made by squaring a whole number is called a perfect square, such as: 16, 25, 36 ...

Numbers	Perfect Square
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169

Try to remember at least the first 10 numbers with their perfect squares.

With your teacher, use the calculator to find the square root.
Does it work?

Numbers	Square Root
0	0
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13

1.6 Roman Numerals

The numbers that we use today are called Arabic numerals. These numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
In the past, people used fingers or objects such as sticks and pebbles for counting, but this was difficult when large numbers were needed.
Different symbols were then invented to represent numbers.
The Romans invented a system for writing numbers. They used symbols to represent numbers.

Arabic numerals	Roman numerals
1	I
2	II
3	III
4	IV
5	V
6	VI
7	VII
8	VIII
9	IX
10	X

You can see that there is no symbol for the number 2 or 4 or 8.
They used the same symbol I or V or X and joined them with each other to represent large numbers.

Examples

A - Write the Roman numeral for 4.

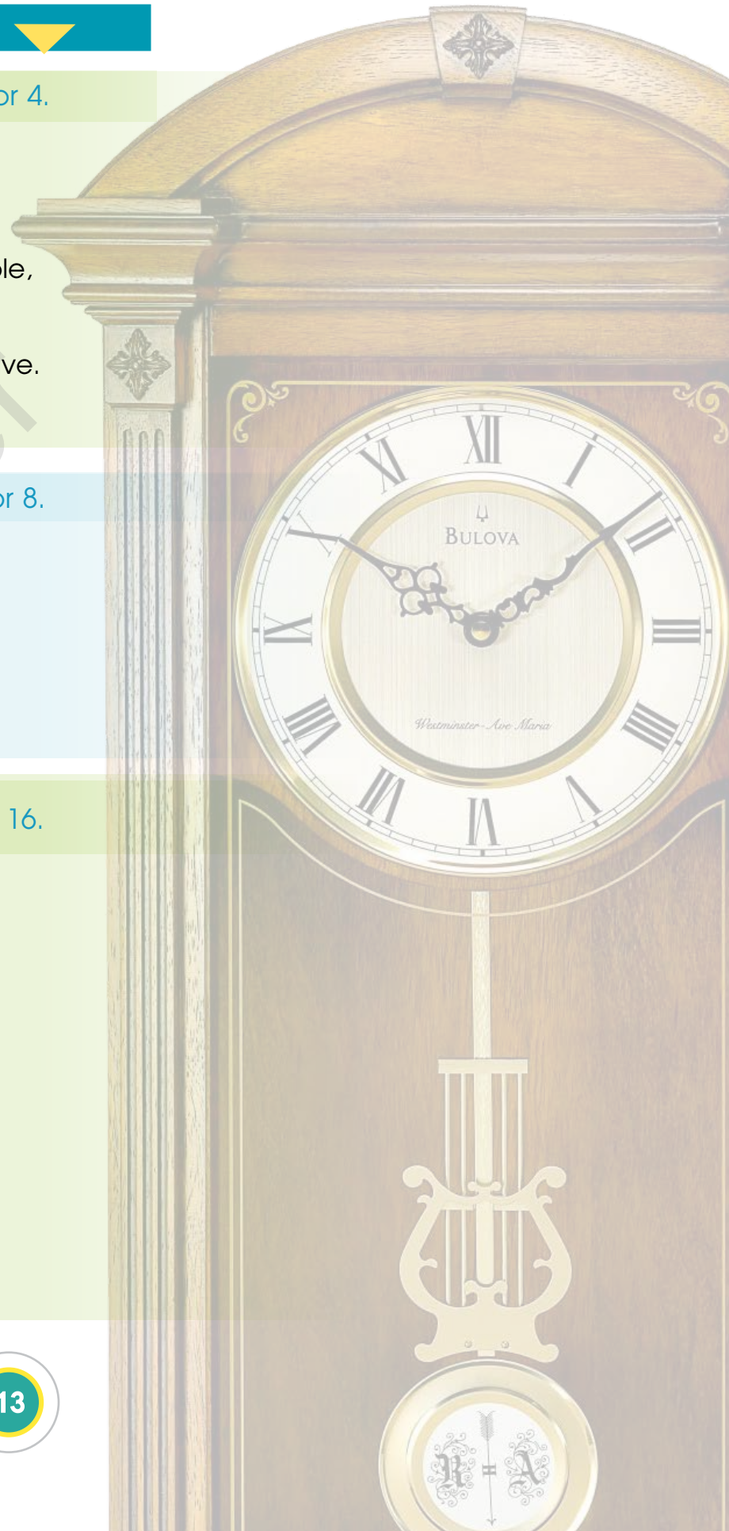
1 = I
So 4 = IIII
But, as you can see in the table,
4 is written as IV = 1 and 5
Therefore IV = is one before five.

B - Write the Roman numeral for 8.

8 = 5 and 3
= V and III
= VIII

C - Write the Roman numeral for 16.

16 = 10 and 5 and 1
= X,V,I
= XVI



Read the table below:

Arabic numeral	Roman numeral	Arabic numeral	Roman numeral
1	I	13	XIII
2	II	14	XIV
3	III	15	XV
4	IV	16	XVI
5	V	17	XVII
6	VI	18	XVIII
7	VII	19	XIX
8	VIII	20	XX
9	IX	50	L
10	X	100	C
11	XI	500	D
12	XII	1000	M

Exercises

1- Write the Roman numerals in the table:

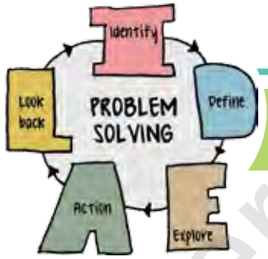
Arabic numeral	1	5	10	20	50	100	500	1000
Roman numeral	I	V	X	XX	L	C	D	M

2- Write the Roman numerals of:

a)	4	IV	b)	8	VIII	c)	11	XI	d)	19	XIX
e)	2	II	f)	14	XIV	g)	7	VII	h)	20	XX

3- Write the Arabic numerals of:

a)	III	3	b)	V	5	c)	M	1000
d)	L	50	e)	XVI	16	f)	XX	20



1.7 Problem Solving

Problem Solving Guide

1- Understand the question.
- What question is the problem asking?

2- Understand word meanings.
- Is there an unfamiliar word in the problem?
- How can I figure out the meaning?

3- Understand how to solve.
- What do I already know?
- How can I use what I know to solve the problem?

4- Solve and check

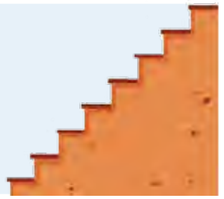
Use the strategies to solve the following problems.

1- a) Arrange the following digits to make the highest number you can:
0, 4, 8, 1, 9 98410

b) Write the number in words.
ninety-eight thousand four hundred ten

c) Arrange the digits to make the smallest number then write it in words.
10489, ten thousand four hundred eighty-nine

2- Help Tom arrange every following set in ascending order.



a) (-521), 521, (-215), 215, (-152), 152

-521, -215, -152, 152, 215, 521

b) 6533, -144, -7859, 12563, 10, -457201, -687

-457201, -7859, -687, -144, 10, 6533, 12563

3- Try to write today's date in Roman numerals.

SOLVE THIS IN CLASS

4- Can you do this simple addition?

MCDXLVI + DLXXVII

SKIP THIS ONE

5- Represent each of the following cases using the right number (Show the positive and negative signs).

a) The temperature in Moscow is twenty five degrees below zero.

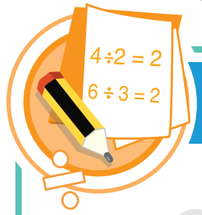
-25 °C

b) Jen baked the cake in her oven at three hundred and two degrees Celsius. 300 °C

c) Jack and John swam in the Dead Sea, whose depth is 298 m below sea level. -298 m

d) Sara borrowed $10 from her sister. $10

e) Alfred gave his son $250 to buy a new tab. $250



2.1 Divisibility

Number 2 can be divided by 1 and 2. These numbers are called the divisors of number 2.

If one number is exactly divisible by another number, the second number is called the divisor of the first number.

Example

Write all divisors of 6.

1 x 6 = 6
2 x 3 = 6
3 x 2 = 6
6 x 1 = 6

or

6 ÷ 1 = 6
6 ÷ 2 = 3
6 ÷ 3 = 2
6 ÷ 6 = 1

The number 6 can be divided by 1, 2, 3 and 6.

Exercise

1- Write divisors of the following:

Number	Divisors
3	1, 3
8	1, 2, 4, 8
12	1, 2, 3, 4, 6, 12
1	1
5	1, 5
9	1, 3, 9

Even Numbers

A number which is divisible by 2 is an **even number**.

For example,

2, 4, 6, 8, 10 ……..

Odd Numbers

A number which is not divisible by 2 is called an **odd number**.

For example,

1, 3, 5, 7, 9 ……..

Exercises

1- List all even numbers below 20 and above (-20). 18, 16, 14, 12, 10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10, -12, -14, -16, -18

2 - List all even numbers greater than 75 and less than 100. 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98

3 - List all odd numbers less than 25 and above (-10). 23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9

4 - List all odd numbers greater than 150 and less than 200. 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199

5 - Tick (✓) in the right place.

Number	50	55	61	63	-70	-75	76	101	-128
Even	✓				✓		✓		✓
Odd		✓	✓	✓		✓		✓	

6- True/False:

a- Every number coming after an even number is an odd number (True).

b- Every number coming after an odd number is an even number (True).

Tests of Divisibility

One whole number is divisible by another if, after dividing, the remainder is zero.

Test of divisibility by 2
If the digit in the ones place of a number is 0, 2, 4, 6, 8, then the number is divisible by 2.
Even numbers are divisible by 2, odd numbers are not.

If one whole number is divisible by another, then the second number is a factor of the first number.

Exercise

Circle the numbers that are divisible by 2.

a) 124 b) 189 c) 335

d) 1002 e) 2023 f) 500

Test of divisibility by 3
If the sum of a number's digits is divisible by 3, then the number is divisible by

Example

Is 234 divisible by 3?

Add the number's digits 2 + 3 + 4 = 9

9 is exactly divisible by 3.

(3 x 3 = 9).

So, 234 is divisible. Check it !

Conclusion: A divisibility test is a rule for determining whether one whole number is divisible by another. It is a quick way to find factors of large numbers. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Exercise

1- Circle the numbers that are divisible by 3.

a) 634	b) 725	c) 3540
d) 2304	e) 3612	f) 2000

Test of divisibility by 4

If the digits at the tens and ones place are 0 or are divisible by 4, then the number is divisible by 4.

Examples

1- Is 1600 divisible by 4?
1600 is divisible by 4, as the tens and ones digits are zero.

2- Is 124 divisible by 4?
124 is divisible by 4 because the tens and ones digits make 24 which is divisible by 4.

Exercise

2- Circle the numbers that are divisible by 4.

a) 348	b) 330	c) 5016
d) 801	e) 24900	f) 12814

Test of divisibility by 6

If the sum of the digits of a number is divisible by 3, and is divisible by 2 at the same time, then the number is divisible by 6.

Example

Is 24 divisible by 6?
24 is an even number so it is divisible by 2.
2 + 4 = 6 which is divisible by 3.
So 24 is divisible by 6.

Exercise

1- Circle the numbers that are divisible by 6.

a) 72	b) 99	c) 108
d) 130	e) 335	f) 300

Test of divisibility by 9

If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Example

Is 459 divisible by 9?
Add 4 + 5 + 9 = 18 (18 is divisible by 9).
so 459 is divisible by 9.

Exercise

2- Circle the numbers that are divisible by 9.

a) 99	b) 182	c) 693	d) 224
e) 23564	f) 9720	g) 8363	h) 23220

Test of divisibility by 5

If the digit at the ones place of a number is 0 or 5, then the number is divisible by 5.

Example

Is 500 divisible by 5?

The digit at the ones place is zero.

So, 500 is divisible by 5.

Test of divisibility by 10

If the digit at the ones place of a number is 0, then the number is divisible by 10.

Example

Is 200 divisible by10?

The digit at the ones place is 0.

So, 200 is divisible by10.

Exercises

1 - List all numbers divisible by 5 below 25. 5, 10, 15, 20

2 - List all numbers divisible by 10 above 10 but below 100. 20, 30, 40, 50, 60, 70, 80, 90

Note: A number that is divisible by 10 is always divisible by 5.

Divisibility Tests	Example
A number is divisible by **2** [if the last digit is 0, 2, 4, 6, or 8]	148 is divisible by 2 since the last digit is 8.
A number is divisible by **3** [if the sum of digits is divisible by 3.]	168 is divisible by 3 since the sum of digits is (15) = (1 + 6 + 8 = 15) and 15 is divisible by 3 = 5.
A number is divisible by **5** if the last digit is either 0 or 5.	325 is divisible by 5 since the last digit is 5.
A number is divisible by **4** if the number formed by the last two digits is divisible by 4.	516 is divisible by 4 since 16 is divisible by 4.
A number is divisible by **6** if it is divisible by 2 and 3.	150 is divisible by 6 since it is divisible by 2 and it is divisible by3.
A number is divisible by **8** if the number formed by the last three digits is divisible by 8.	120 is divisible by 8 since 120 is divisible by 8.
A number is divisible by **9**, if the sum of the digits is divisible by 9.	549 is divisible by 9 since the sum of the digits is (18) and 18 is divisible by 9.
A number is divisible by **10** if the last digit is 0.	2340 is divisible by 10 since the last digit is 0.

2.2 Factors

Look at these apples.
They are 8 apples.

Arrange them into equal groups in different ways.

Groups of one: Eight groups, every group contains one apple.

1 x 8 = 8

Groups of two: Four groups, every group contains two apples.

2 x 4 = 8

Groups of four: Two groups, every group contains four apples.

4 x 2 = 8

Groups of eight: One group contains eight apples.

8 x 1 = 8

You will see that each time you arrange the apples in equal groups, none is left over.

①, ②, ④, and ⑧ are called 'factors' of 8.

We can find factors of numbers by arranging them in groups.

Number	Factors	
4	① x 4	② x 2 ④ x 1 — 1, 2, 4
6	① x 6	② x 3 ③ x 2 ⑥ x 1 — 1, 2, 3, 6
12	① x 12	② x 6 ③ x 4 ④ x 3 ⑥ x 2 ⑫ x 1 — 1, 2, 3, 4, 6, 12

Look at the factors again.

You will see that 1 is a factor of every number.

You will also see that each number is a factor of itself.

When a number is divided by any of its factors, there is **no** remainder.

The factors of 8 are : 1, 2, 4, and 8.

Exercises

Find the factors of the following numbers.

a) 9 1, 3, 9	b) 12 1, 2, 3, 4, 6, 12	c) 16 1, 2, 4, 8, 16	d) 26 1, 2, 13, 26
e) 28 1, 2, 4, 7, 14, 28	f) 32 1, 2, 4, 8, 16, 32	g) 40 1, 2, 4, 5, 8, 10, 20, 40	h) 75 1, 3, 5, 15, 25, 75

2. 3 Prime and Composite Numbers

Look at the table:

Number	1	5	12
Factors	1	1, 5	1, 2, 3, 4, 6, 12

From the table we can see that:
5 has two factors: (1, 5). So, 5 is a **prime number.**

12 has more than two factors. So, 12 is a **composite number.**

Numbers that have exactly two factors "one and the number itself" are called '**prime**' numbers.
2, 3, 5, 7, 11, 13, 17, 19, etc... are prime numbers.

Numbers that have more than two factors are called '**composite**' numbers.
4, 6, 8, 10, 12, 14, 15, 16, 18, 20, etc, are composite numbers.

Exercises

- Define a **prime** number in your words.
- Is there a relation between **odd** numbers and **prime** numbers?
- Is there a **prime and even** number at the same time?

1 - Circle the prime numbers.

15 19 28 37 47 49 29
7 9 11 13 53 23 25

2 - Circle the composite numbers.

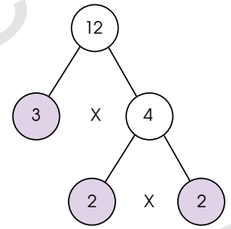
4 12 13 15 17 19 21
23 25 27 29 30 43 680

3 - Write **T** for true or **F** for false.

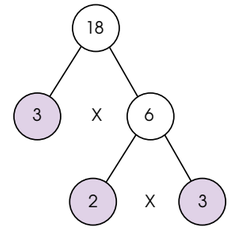
a) 4 is the smallest prime number. F
b) 3 is the smallest composite number. F
c) 7 is the only prime number greater than 5 and less than 11. T
d) 73 is a prime number. T
e) All even numbers greater than 2 are composite. T
f) All odd numbers are prime numbers. F

Prime Factors

Let's find the factors of 12 by using a 'factor tree'.



The factors of 18 are:

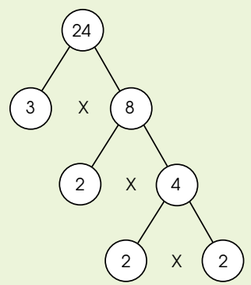


The numbers inside the coloured circles are called' **prime factors**'. We cannot do any more factors of them. Numbers 2 and 3 they cannot be divided by other numbers than one and themselves. The method by which we find prime factors of a number is called '**prime factorization**'.

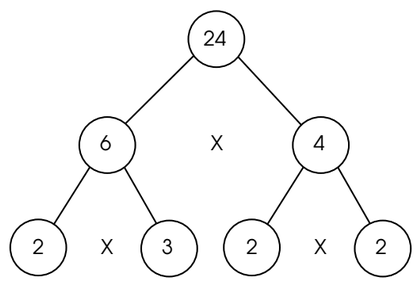
We can make a factor tree in different ways but the prime factors at the end will always be the same.

Example

Find the prime factors of 24



The prime factors of 24 are:
3 x 2 x 2 x 2

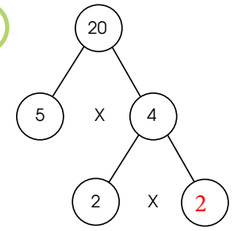


The prime factors of 24 are:
2 x 3 x 2 x 2

Exercises

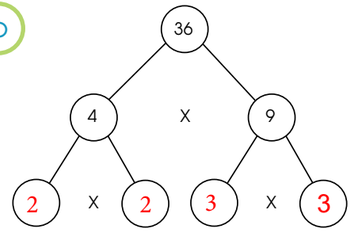
1- Complete the factor trees.

a



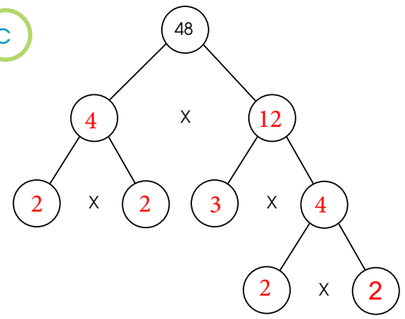
Prime factors: 5 x 2 x 2

b



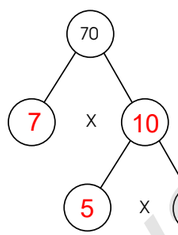
Prime factors: 3 * 2 * 2 * 3

c



Prime factors: 2 * 2 * 3 * 2 * 2

d



Prime factors: 7 * 5 * 2

2- Write the prime factors of the following numbers.

a) 10 — 2 * 5

b) 64 — 2 * 2 * 2 * 2 * 2 * 2

c) 100 — 5 * 2 * 5 * 2

d) 72 — 2 * 3 * 3 * 2 * 2

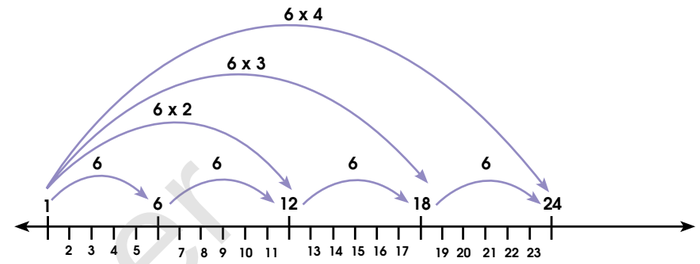
e) 108 — 3 * 3 * 3 * 2 * 2

f) 125 — 5 * 5 * 5

2.4 Multiples

Look at the multiplication table given below:

6 x 1 = 6
6 x 2 = 12
6 x 3 = 18
6 x 4 = 24
6 x 5 = 30



6, 12, 18, 24 and 30 are called the '**multiples**' of 6.
When you multiply numbers 1, 2, 3, 4, 5, ... by the number 6, you get the multiples of the number 6.

Example

Find the multiples of 2, 3, 4 and 5

a) Multiples of 2 are: 2, 4, 6, 8, 10, ...
b) Multiples of 3 are: 3, 6, 9, 12, 15, ...
c) Multiples of 4 are: 4, 8, 12, 16, 20, ...
d) Multiples of 5 are: 5, 10, 15, 20, 25, ...

Exercises

1- Write the first four multiples of:

a)	3	6	9	12	15
b)	8	16	24	32	40
c)	9	18	27	36	45
d)	10	20	30	40	50
e)	15	30	45	60	75
f)	25	50	75	100	125

Look at the multiples below:

56 = 7 x 8 — 56 is a multiple of 7. 56 is also a multiple of 8.

45 = 9 x 5 — 45 is a multiple of 9. 45 is also a multiple of 5.

2 - Write the first three multiples of the following numbers.

a) 11: 11, 22, 33

b) 12: 12, 24, 36

c) 14: 14, 28, 42

d) 31: 31, 62, 93

3- Circle all the multiples of (5).

a) 13 b) 17 c) (25) d) (135) e) (180) f) 149

4- Circle all the multiples of (7).

a) (14) b) (70) c) 34 d) 100 e) 55 f) (112)

Common Multiples

Multiples of 6 are: 6, 12, (18), 24, 30, (36),.....

Multiples of 9 are: 9, (18), 27, (36), 45, 54,.....

We see that 18 and 36 are multiples of both 6 and 9.

So 18 and 36 are called the '**common multiples**' of 6 and 9.

Example

Find the 1st common multiple of 2 and 3.

Multiples of 2 are: 2, 4, (6), 8, 10

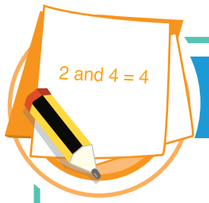
Multiples of 3 are: 3, (6), 9, 12, 15

1st common multiple of 2 and 3 is 6.

Exercise

Write the first five multiples of the following and pick out the common multiples.

Numbers		Multiples	Common multiples
a)	2	10 ,8 ,(6) ,4 ,2	6
	3	15 ,12 ,9 ,(6) ,3	
b)	3	3, 6, 9, 12, 15	15
	5	5, 10, 15, 20, 25	
c)	6	6, 12, 18, 24, 30	12, 24
	12	12, 24, 36, 48, 60	
d)	8	8, 16, 24, 32, 40	None
	7	7, 14, 21, 28, 35	



2.5 Least Common Multiple (LCM)

Multiples of 3 are 3, 6, 9, 12, (15), 18, 21, 24, 27, (30)
Multiples of 5 are 5, 10, (15), 20, 25, (30), 35, 40, 45, 50.
The smallest of the common multiples are 15 and 30.

The least common multiple LCM is 15,
because 15 is is the first common multiple between two numbers.

Example

Find the LCM of 2 and 3.

Numbers	Multiples	Common multiples	LCM
2	2, 4, (6), 8, 10, (12), 14, 16, (18), 20	6, 12 and 18	6
3	3, (6), 9, (12), 15, (18), 21, 24, 27, 30		

Exercise

Find the LCM of:

Numbers	Multiples	Common multiples	LCM
2	2, 4, 6, 8, 10, 12, 14, 16, 18, 20	6, 12, 18	6
3	3, 6, 9, 12, 15, 18, 21, 24, 27, 30		
6	6, 12, 18, 24, 30, 36, 42, 48, 54, 60	18, 36, 54	18
9	9, 18, 27, 36, 45, 54, 63, 72, 81, 90		
5	5, 10, 15, 20, 25, 30, 35, 40, 45, 50	40	40
8	8, 16, 24, 32, 40, 48, 56, 64, 72, 80		

We can find the LCM of two numbers by prime factorization and division methods. First we find the prime factors of the numbers, and then we find the LCM by multiplying together all the prime factors.

Examples

1- Find LCM of **12** and **18** using the prime factorization method.
First we find the prime factors.

2	**12**
2	6
3	3
	1

2	**18**
3	9
3	3
	1

Prime factors of **12** = (2) x 2 x (3)
Prime factors of **18** = (2) x 3 x (3)
The common factors = (2) and (3)
The uncommon factors = 2 and 3

Common factors are included only once

By multiplying the common and uncommon factors we get

= 2 x 3 x 2 x 3
LCM = 36

2- Find LCM of 25 and 30.

prime factors of 25 = 5 x (5)
prime factors of 30 = 2 x 3 x (5)
5 x 3 x 5 = 150

The lowest common multiple of 25 and 30 is 150.

We can also find LCM by dividing 12 and 18 together by their common multiple.

2	12, 18
2	6, 9
3	3, 9
3	1, 3
	1, 1

First divide by the smallest number and then go on to the greatest number.

Then we multiply all the factors.

= 2 x 2 x 3 x 3

= 36

So 36 is the LCM of 12 and 18

Exercise

Find the LCM of the following:

a) 20 and 30

2	20, 30
5	10, 15
2	2, 3
3	1, 3
	1, 1

2 * 5 * 2 * 3 = 60

b) 45 and 90

3	45, 90
5	15, 30
3	3, 6
2	1, 2
	1, 1

3 * 5 * 3 * 2 = 90

c) 24 and 32

2	24, 32
2	12, 16
2	6, 8
2	3, 4
2	3, 2
3	3, 1
	1, 1

2 * 2 * 2 * 2 * 2 * 3 = 96

d) 18 and 24

2	18, 24
3	9, 12
2	3, 4
2	3, 2
3	3, 1
	1, 1

2 * 3 * 2 * 2 * 3 = 72

LCM of Three Numbers

The LCM of three numbers can also be found by prime factorization and division methods.

Example

Find the LCM of **6**, **12** and 18.
First, use the prime factorization method.

2	**6**
3	3
	1

2	**12**
2	6
3	3
	1

2	**18**
3	9
3	3
	1

Prime factors of **6** = (2) x (3)

Prime factors of **12** = (2) x 2 x (3)

Prime factors of **18** = (2) x 3 x (3)

Common factors = (2) and (3)

Uncommon factors = 2 and 3

LCM = 2 x 3 x 2 x 3

= 36

LCM of **6, 12, 18** by division method is:

2	**6, 12, 18**
2	3, 6, 9
3	3, 3, 9
3	1, 1, 3
	1, 1, 1

LCM = 2 x 2 x 3 x 3

= 36

Exercises

1 - Find the LCM by division method.

a) 2, 4, 6

2	2	4	6
2	1	2	3
3	1	1	3
	1	1	1

2 * 2 * 3 = 12

b) 16, 32, 64

2	16	32	64
2	8	16	32
2	4	8	16
2	2	4	8
2	1	2	4
2	1	1	2
	1	1	1

2* 2* 2 * 2 *2 * 2= 64

c) 3, 6, 12

2	3	6	12
2	3	3	6
3	3	3	3
	1	1	1

3 * 2 * 2 = 12

d) 7, 14, 28

2	7	14	28
2	7	7	14
7	7	7	7
	1	1	1

7 * 2 * 2 = 28

2 - Find the LCM by prime factorization method.

a) 8, 16 and 32

2 * 2 * 2 *2 * 2 = 32

b) 15, 25 and 30

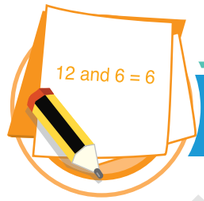
2 * 3 * 5 * 5 = 150

c) 10, 15 and 20

2 * 2 * 3 * 5 = 60

d) 8, 12 and 16

2 * 2 * 2 * 2 * 3 = 48



2.6 Greatest Common Factor (GCF)

We know that a divisor is a number which divides another number.

One number can have many divisors, and each divisor is a factor of that number.

The factors of **12** are: 1, 2, 3, 4, 6, 12

The factors of **18** are: 1, 2, 3, 6, 9, 18

The common factors of **12** and **18** are: 1, 2, 3, and 6

The greatest or highest common factor GCF is 6.
It means that 6 is the greatest number that can divide 12 and 18.

Exercise

Complete the table to find the GCF.

Find all the factors for each number.

Circle the common factors.

Choose the greatest of those.

Numbers	Factors	Common factors	GCF
15	15 , 5 , 3 , 1	1,5	5
20	20 , 10 , 5 , 4 , 2 , 1		
10	1, 2, 5, 10	1, 2, 5, 10	10
40	1, 2, 4, 5, 8, 10, 20, 40		
15	1, 3, 5, 15	1, 15	5
25	1, 5, 25		

It is easy to find the GCF of smaller numbers by using the prime factorization method.

Example

Find the GCF of 18 and 28.

2	18
3	9
3	3
	1

2	**28**
2	14
7	7
	1

Prime factors of 18 = (2) x 3 x 3

Prime factors of 28 = (2) x 2 x 7

Common factor = 2

GCF = 2

Sometimes, it is difficult to find the GCF of large numbers by using the prime factorization method so we use long division method.

Example

Find the GCF of 186 and 252.

First, the greater number by the smaller number.

```
      1
186 ) 252
    - 186
      66   (remainder)
```

Make the remainder the divisor of the first divisor.

```
      2
66 ) 186
    - 132
      54   (remainder)
```

Make the remainder the divisor of the second divisor.

```
      1
54 ) 66
    - 54
      12   (remainder)
```

Make the remainder the divisor of the third divisor.

```
      4
12 ) 54
    - 48
      6    (remainder)
```

Make the remainder the divisor of the fourth divisor.

```
      2
(6) 12
    - 12
      0    (remainder)
```

The last divisor is 6.

So, the GCF of 186 and 252 is 6.

Exercises

1 - Find the GCF by using prime factorization method.

a) 30, 45	b) 42, 70	c) 24, 36	d) 60, 96
15	14	12	12

2 - Find the GCF by using long division method.

a) 72, 162	b) 220, 286	c)128, 112	d) 92, 276
18	22	16	0

GCF of Three Numbers

We can find the GCF of three numbers by using prime factorization method. First, we find the prime factors, then we find the GCF by multiplying all common factors.

Example

Find the GCF of 18, 24 and 48.
First, we find their prime factors.

2	18
3	9
3	3
	1

2	24
2	12
2	6
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

prime factors of 18 = ② x 3 x ③
prime factors of 24 = ② x 2 x 2 x ③
prime factors of 48 = ② x 2 x 2 x 2 x ③
common factors = 2 x 3
GCF = 6

Exercise

Find the GCF by using prime factorization method.

a) 56, 84, 70 — 2 * 7 = 14
b) 30, 36, 42 — 2 * 3 = 6
c) 30, 50, 140 — 2 * 5 = 10
d) 60, 80, 100 — 2 * 2 * 5 = 20

We can also find GCF of three numbers by using the long division method. First we find the GCF of two numbers in the same way by dividing the greater number by the smaller one. Then we divide the third number by the GCF of the first two numbers.

Example

Find the GCF of 128, 112 and 80.

First, we divide 128 by 112.

```
    1
112 | 128
  - 112
    16
```

```
    7
16 | 112
  - 112
     0
```

GCF of 128 and 112 is 16

Now, we divide 80 by 16.

```
    5
16 | 80
  - 80
     0
```

The GCF of 16 and 80 is 16,
so the GCF of 128, 112 and 80 is 16.

Exercise

Find the GCF by using long division method.

a) 76, 171, 285 — 19
b) 52, 72, 84 — 4
c) 485, 515, 855 — 5
d) 34, 85, 170 — 17

2.7 Problem Solving

Remember

1- Read
2- Plan
3- Solve
4- Check

1. A radio station is having a promotion in which every 12th caller receives a free concert ticket and every 15th caller receives a limo ride. Which caller will be the first one to win both? 60th

2. Cups are sold 6 to a package and plates are sold 12 to a package. If you want to have the same number of each item for a party, what is the least number of packages of each you need to buy? 2 cups to a package 1 plate to a package

3. Tony needs to ship 24 comedy DVDs, 48 animated DVDs, and 30 musical DVDs. He can pack only one type of DVD in each box and he must pack the same number of DVDs in each box. What is the greatest number of DVDs Tony can pack in each box? 6

4. A full moon occurs every 30 days. If the last full moon occurred on a Monday, how many days will pass before a full moon occurs again on a Monday?

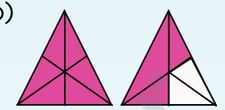
5. Mei has 15 oranges, 9 peaches and 18 pears. She wants to put all of the fruit into baskets with each basket having the same number of pieces of fruit in it. Without mixing the fruit, what is the greatest number of pieces of fruit Mei can put in each basket? 3

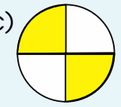
3.1 Fraction Review

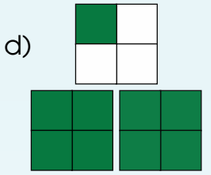
Revision

1- Use a fraction to represent the shaded part of each figure.

a)  $\frac{2}{5}$

b)  $1\frac{3}{5}$

c)  $\frac{2}{4}$

d)  $2\frac{1}{4}$

2- Complete to make equivalent fractions.

a) $\frac{4}{5} = \frac{20}{25}$ b) $\frac{44}{77} = \frac{4}{7}$ c) $\frac{63}{72} = \frac{7}{8}$ d) $\frac{25}{40} = \frac{5}{8}$

3- Change into compound fractions.

a) $\frac{25}{6}$ $4\frac{1}{6}$

b) $\frac{52}{6}$ $8\frac{4}{6}$

c) $\frac{54}{5}$ $10\frac{4}{5}$

d) $\frac{37}{7}$ $5\frac{2}{7}$

4- Change into improper fractions.

a) $2\frac{1}{6}$ $\frac{13}{6}$

b) $15\frac{1}{5}$ $\frac{76}{5}$

c) $4\frac{2}{4}$ $\frac{18}{4}$

d) $4\frac{2}{15}$ $\frac{62}{15}$

Remember

You know that

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

Multiply the numerator and the denominator by the same number.

We can see that $\frac{1}{2}$ is the simplest form of $\frac{4}{8}$.

We can reduce $\frac{4}{8}$ to $\frac{1}{2}$ by dividing both 4 and 8 by 4.

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Divide the numerator and the denominator by the same number

Another method of reducing fractions to their simplest form is to find a common factor for the numerator and the denominator.

Divide them by the factor.

Example

Reduce $\frac{20}{40}$ to its simplest form.

$$\frac{\overset{10}{\cancel{20}}}{\underset{20}{\cancel{40}}} = \frac{\overset{5}{\cancel{10}}}{\underset{10}{\cancel{20}}} = \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}} = \frac{1}{2}$$

(dividing by 2) (dividing by 2) (dividing by 5)

Exercises

1- Write an equivalent form to the fraction.

a) $\frac{8}{12} = \frac{2}{3}$

b) $\frac{27}{36} = \frac{3}{4}$

c) $\frac{28}{35} = \frac{4}{5}$

d) $\frac{10}{18} = \frac{5}{9}$

2- Reduce the fractions to their simplest form.

a) $\frac{14}{36} = \frac{7}{18}$

b) $\frac{6}{20} = \frac{3}{10}$

c) $\frac{6}{24} = \frac{1}{4}$

d) $\frac{-18}{35} = \frac{-18}{35}$

e) $\frac{28}{40} = \frac{7}{10}$

f) $\frac{18}{12} = \frac{3}{2}$

g) $\frac{27}{36} = \frac{3}{4}$

h) $\frac{28}{35} = \frac{4}{5}$

i) $\frac{-10}{18} = \frac{-5}{9}$

3.2 Equivalent Fraction

An equivalent fraction seems different but gives exactly the same value, you can make equivalent fractions by multiplying the numerator and denominator together by the same number.

$\frac{2 \times 2}{5 \times 2}$ =  $\frac{4 \times 2}{10 \times 2}$ =  $\frac{8}{20}$

You can simplify the fractions by dividing the numerator and denominator by the the same number, this is called cancelling.

 $\frac{9 \div 3}{30 \div 3} = \frac{3}{10}$ 

Exercises

Complete the equivalent fractions.

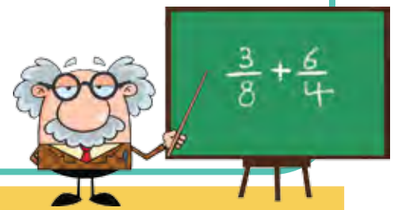
a) $\frac{7}{9} = \frac{\boxed{35}}{45} = \frac{\boxed{49}}{63}$

b) $\frac{1}{2} = \frac{7}{\boxed{14}} = \frac{\boxed{6}}{12}$

c) $\frac{2}{3} = \frac{\boxed{20}}{30} = \frac{\boxed{8}}{12}$

d) $\frac{-50}{100} = \frac{-10}{\boxed{20}} = \frac{\boxed{-5}}{10}$

e) $\frac{1}{5} = \frac{7}{\boxed{30}} = \frac{2}{\boxed{10}}$



3.3 Addition/Subtraction Fractions

1- Add/subtract fractions (**like denominators)**.

a) $\frac{3}{5} + \frac{3}{5} = \frac{3+3}{5} = \frac{6}{5}$

b) $\frac{12}{20} - \frac{3}{20} = \frac{9}{20}$

c) $\frac{3}{14} + \frac{5}{8} = \frac{\boxed{94}}{\boxed{112}}$

d) $\frac{3}{7} - \frac{\square}{\square} = 1$

e) $\frac{6}{\boxed{8}} + \frac{\boxed{1}}{8} = \frac{7}{8}$

f) $\frac{8}{12} - \frac{2}{\boxed{12}} = \frac{\boxed{6}}{12}$

2- Add mixed numbers (**like denominators**).

a) $3\frac{2}{5} + 3\frac{1}{5} = 6\frac{3}{5}$

b) $6\frac{6}{12} + 4\frac{9}{12} = 11\frac{3}{12}$

c) $3\frac{2}{3} + 7\frac{2}{3} = \boxed{11}\frac{\boxed{1}}{\boxed{3}}$

d) $10\frac{1}{2} + \boxed{7}\frac{\boxed{1}}{\boxed{2}} = 18$

e) $5\frac{8}{9} - 4\frac{7}{9} = \boxed{1}\frac{\boxed{1}}{\boxed{9}}$

f) $7\frac{6}{14} - 3\frac{10}{14} = \boxed{3}\frac{\boxed{5}}{7}$

3- Add unlike **fractions**.

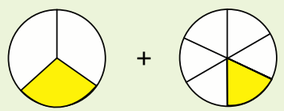
Step1: Remember to simplify equivalent fractions (multiply, divide) to make the bottoms (denominators) similar in both/all fractions.

Step 2: Add the top numbers (the numerators), put the answer over the denominator.

Step 3: Simplify the fraction if needed.

Example

$\frac{1}{3} + \frac{1}{6}$



First of all you should take a look at the bottoms. Notice that they are not equal, so, we will make them equal. The question is what is the simplest way to change 3 to 6 or change 6 to 3? Multiply the top and bottom of ($\frac{1}{3}$) by 2 .

$\frac{1}{3} \overset{\times 2}{=} \frac{2}{6}$ (x 2 below)

Now, the fractions have the same denominators which is equal to 6. Now add them together.

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

If we need the simplest form $\frac{3}{6} = \frac{1}{2}$

Multiply common fractions by a whole number.

Exercises

1- Find the product.

a) $5 \times \frac{1}{3} = \frac{5 \times 1}{3} = \frac{5}{3}$

b) $3 \times \frac{3}{5} = \frac{9}{5}$

c) $3 \times 4\frac{3}{5} = \frac{69}{5}$

$3 * \frac{23}{5} = \frac{69}{5}$

d) $5 \times \frac{6}{9} = \frac{30}{9} = \frac{10}{3}$

e) $6 \times 2\frac{3}{5} = \frac{78}{5}$

f) $15 \times \frac{3}{5} = \frac{45}{5} = 9$

g) $7 \times 3\frac{2}{3} = \frac{77}{3}$

Hint: To find $3 \times 3\frac{1}{3}$

Remember that: $3\frac{1}{3} = 3 + \frac{1}{3}$

So $= 3 \times (3 + \frac{1}{3})$

$= 3 \times 3 + 3 \times \frac{1}{3}$

$= 9 + 1$

$= 10$

2- Add and subtract.

a) $\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

b) $2\frac{3}{4} + 5\frac{1}{4} = \frac{33}{4}$

c) $\frac{2}{4} + \frac{3}{8} = \frac{7}{8}$

d) $4\frac{1}{8} + 3\frac{2}{8} = \frac{59}{8}$

e) $\frac{7}{12} + \frac{5}{36} = \frac{26}{36}$

f) $2\frac{1}{3} + 2 = \frac{13}{3}$

g) $\frac{7}{9} - \frac{1}{3} = \frac{4}{9}$

h) $3 - \frac{1}{4} = \frac{11}{4}$

3.4 Multiply Two Fractions

Multiply the **numerators together** and the **denominators together** too.

Then simplify the fraction if needed.

Example

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$$

Exercises

1- Find the product of the following.

a) $\frac{1}{4} \times \frac{1}{4} =$ $\frac{1}{16}$

b) $\frac{2}{5} \times \frac{7}{6} =$ $\frac{14}{30}$

c) $\frac{3}{8} \times \frac{3}{9} =$ $\frac{9}{72}$

d) $7 \times \frac{7}{100} =$ $\frac{49}{100}$

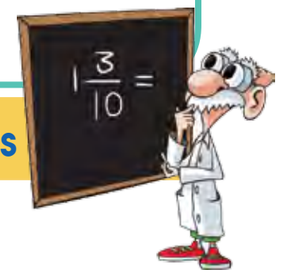
2- Complete the multiplication sentence.

a) $\frac{2}{6} \times \boxed{\frac{1}{5}} = \frac{2}{30}$

b) $\frac{2}{11} \times \boxed{\frac{2}{11}} = \frac{4}{22}$

c) $3 \times \frac{3}{4} = \frac{1}{4} \times \boxed{\frac{9}{4}}$ $\quad \frac{9}{4} = \frac{1}{4} * \frac{9}{4}$

3.5 Multiply With Mixed Numbers



$$4\frac{1}{2} \times 2\frac{1}{3}$$

Convert the mixed number to an improper fraction.

$$4\frac{1}{2} = \frac{9}{2} \qquad 2\frac{1}{3} = \frac{7}{3}$$

$$\frac{9}{2} \times \frac{7}{3} = \frac{63}{6}$$

Remember

To convert a mixed fraction to an improper fraction:

1- Multiply the whole number by the fraction's denominator.

2- Add that to the numerator.

3- Write the result on top of the original denominator.

Exercise

Find the product.

a) $1\frac{1}{2} \times 2\frac{1}{4} =$ $\frac{3}{2} * \frac{9}{4} = \frac{27}{8}$

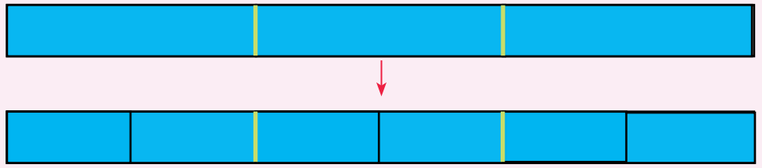
b) $3\frac{1}{3} \times 4\frac{1}{2} =$ $\frac{10}{3} * \frac{9}{2} = \frac{90}{6}$

c) $\frac{7}{8} \times 2\frac{3}{5} =$ $\frac{7}{8} * \frac{13}{5} = \frac{91}{40}$

3.6 Fraction Division

Divide a whole number by a fraction.

$3 \div \frac{1}{2} = \_\_\_$



We get 6 pieces

Exercise

1- Find.

a) $2 \div \frac{1}{5} = \frac{2}{1} * \frac{5}{1} = 10$

b) $3 \div \frac{1}{3} = \frac{3}{1} * \frac{3}{1} = 9$

c) $5 \div \frac{1}{4} = \frac{5}{1} * \frac{4}{1} = 20$

Divide a fraction by a whole number and divide a fraction by a fraction.

First of all, we have to know what is the reciprocal of (fraction, whole number).

To find the reciprocal, simply switch between numerator and denominator. The reciprocal of $\frac{4}{1}$ is $\frac{1}{4}$ and the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Now to find the answer for the division of a fraction by a whole number or a fraction by a fraction look at the next example.

Dividing here is the same as multiplying the reciprocal of a fraction by the other fraction.

So: $\frac{2}{3} \div \frac{4}{3} = \frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$

2- Find the reciprocal.

a) $\frac{2}{3} = \frac{3}{2}$

b) $5 = \frac{1}{5}$

c) $\frac{11}{130} = \frac{130}{11}$

d) $\frac{-2}{4} = \frac{-4}{2}$

3- Find the following.

a) $\frac{3}{4} \div 6 = \frac{1}{8}$

b) $\frac{8}{10} \div \frac{2}{5} = 2$

c) $\frac{8}{3} \div \frac{7}{1} = \frac{8}{21}$

d) $\frac{5}{6} \div \frac{5}{10} = \frac{5}{3}$

Think $\frac{4}{6} \div \frac{2}{3} = \frac{\frac{4}{6}}{\frac{2}{3}} = \frac{4 \div 2}{6 \div 3} = \frac{2}{2} = 1$ Is it right?

3.7 Division With Mixed Numbers

$5\frac{3}{8} \div 2\frac{1}{3} =$

Step1: Change to improper fraction.

$5\frac{3}{8} = \frac{43}{8}$, $2\frac{1}{3} = \frac{7}{3}$

Step 2: Write a new division problem with the improper fractions.

$\frac{43}{8} \div \frac{7}{3} = \frac{43}{8} \times \frac{3}{7} = \frac{129}{56}$

Exercises

Divide the following. Simplify the answer.

a) $1\frac{3}{6} \div 2\frac{1}{4} = \frac{9}{6} * \frac{4}{9} = \frac{36}{54} = \frac{2}{3}$

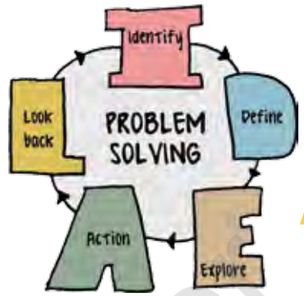
b) $\frac{7}{8} \div 3\frac{2}{4} = \frac{7}{8} * \frac{4}{14} = \frac{28}{112} = \frac{1}{4}$

c) $5\frac{3}{8} \div \frac{1}{3} = \frac{43}{8} * \frac{3}{1} = \frac{129}{8}$

d) $2\frac{7}{5} \div 3\frac{1}{2} = \frac{17}{5} * \frac{2}{7} = \frac{34}{35}$

e) $7 \div 5\frac{3}{8} = 7 * \frac{8}{43} = \frac{56}{43}$

f) $\frac{2\frac{1}{7} \div 8}{56} = \frac{\frac{15}{7} * \frac{1}{8} = \frac{15}{56}}{\frac{15}{56} * 56 = 15}$



3.8 Problem Solving

Remember

1- Read
2- Plan
3- Solve
4- Check

1- You walk $\frac{4}{10}$ of a mile to your friend's house, and then $\frac{6}{10}$ of a mile to school. How far did you walk altogether? $\frac{4}{10} + \frac{6}{10} = \frac{10}{10} = 1$

2- After a party, $\frac{5}{8}$ of the cake is left over. That night, brother ate $\frac{2}{8}$ of the cake. How much is left over after that? $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

3- You go out for a long walk. You walk $\frac{3}{8}$ mile and then sit down to take a rest. Then you walk $\frac{1}{4}$ of a mile. How far did you walk altogether? $\frac{3}{8} + \frac{1}{4} = \frac{5}{8}$

4- A cake recipe requires $1\frac{2}{3}$ cup of sugar for the frosting and $\frac{2}{3}$ cup of sugar for the cake. How much sugar is that altogether?

$1\frac{2}{3} + \frac{2}{3} =$

$\frac{5}{3} + \frac{2}{3} = \frac{7}{3}$

Decimal Fractions

Unit 4

4.1 Decimal Review

Changing common fractions into decimal fractions.

We learned before how to change common fractions into decimal fractions.

- Change into decimal fractions.

a) $\frac{5}{10} = \underline{0.5}$ b) $\frac{69}{100} = \underline{0.69}$ c) $\frac{47}{1000} = \underline{0.047}$ d) $\frac{138}{1000} = \underline{0.138}$

- Change into common fractions.

a) $0.9 = \frac{9}{10}$ b) $0.07 = \frac{7}{100}$ c) $0.10 = \frac{1}{10}$

d) $0.013 = \frac{13}{1000}$ e) $0.009 = \frac{9}{1000}$ f) $0.213 = \frac{213}{1000}$

We also learned how to write a compound fraction as a decimal fraction.

$2\frac{3}{10} = 2.3$ $11\frac{1}{100} = 11.01$

Exercise

Write the following mixed numbers as decimal fractions.

a) $3\frac{1}{10}$ = 3.1 b) $14\frac{3}{100}$ = 14.03 c) $12\frac{11}{100}$ = 12.11 d) $23\frac{9}{1000}$ = 23.009

e) $\frac{45}{10}$ = 4.5 f) $\frac{251}{10000}$ = 0.0251 g) $\frac{100}{1000}$ = 0.1 h) $\frac{7}{1000}$ = 0.007

4.2 Add - Subtract Decimal Numbers

1.431 + 2.32 = ...

Step1: Write down the numbers one under the other (**decimal point lined up**).

Step 2: Put in zeros so the numbers have the same length.

Step 3: Add/subtract

$$\begin{array}{r} 1.431 \\ +\ 2.32 \\ \hline \end{array} \qquad \begin{array}{r} 1.431 \\ +\ 2.320 \\ \hline \end{array} \qquad \begin{array}{r} 1.431 \\ +\ 2.320 \\ \hline 3.751 \end{array}$$

Exercises

Complete the following operations:

a) 7.387 - 1.251 = 6.136 $\quad \begin{array}{r} 7.387 \\ -\ 1.251 \\ \hline 6.136 \end{array}$

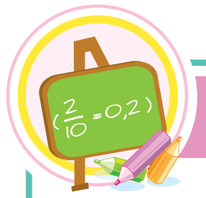
b) 6.222 + 7.2 = 13.422 $\quad \begin{array}{r} 6.222 \\ +\ 7.2 \\ \hline 13.422 \end{array}$

c) 5 + 3.13 = 8.13

d) 0.96 + 1.321 = 2.281

e) 10.777 - 1.112 = 9.665

f) 7.832 - 7.811 = 0.021



4.3 Changing Fractions into Decimals

We already know how to write a fraction with denominators of 10, 100 or 1000. But how do we change a fraction with denominators of 2, 3, 4, 5, 6, 8, 12 or 20 into a decimal fraction?

Remember

$\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$, $\frac{1}{1000} = 0.001$, $\frac{17}{10} = 1.7$ and $\frac{34}{100} = 0.34$

We must change the fraction into an equivalent fraction, with the denominator as a multiple of 10.

- Change $\frac{1}{2}$ into a decimal fraction.

$$\frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$$

Change the fraction to an equivalent fraction with the denominator as a multiple of 10.

- Change $\frac{1}{4}$ into a decimal fraction.

$$\frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$$

Exercises

1- Change the following into decimal fractions.

a) $\frac{931}{100} = 9.31$ b) $\frac{1}{5} = 0.2$ c) $\frac{3}{4} = 0.75$

d) $\frac{65}{25} = 2.56$ e) $\frac{16}{20} = 0.8$ f) $\frac{17}{50} = 0.34$

A mixed number can also be changed into a decimal fraction in the same way.

Example

Change $3\frac{1}{4}$ into a decimal fraction.

$$3 + \frac{1}{4} = 3 + \frac{1 \times 25}{4 \times 25} = 3 + \frac{25}{100}$$

$$= 3 + 0.25 = 3.25$$

2- Write each of the following as a decimal fraction.

a) $3\frac{3}{4} = \frac{15}{4} = 3.75$

b) $8\frac{7}{10} = \frac{87}{10} = 8.7$

c) $4\frac{6}{25} = \frac{106}{25} = 4.24$

d) $6\frac{1}{4} = \frac{25}{4} = 6.25$

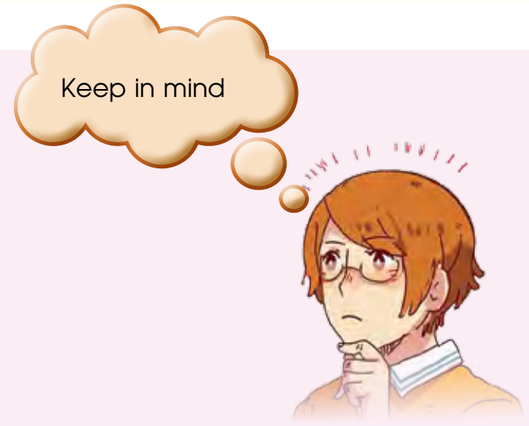
$\frac{2}{4} = 0.5$

$\frac{1}{4} = 0.25$

$\frac{1}{3} = 0.333$

$\frac{3}{4} = 0.75$

$\frac{4}{5} = 0.8$



Changing Decimal Fractions to Their Simplest Fraction

We can reduce a decimal fraction to its simplest form. First we change it into a fraction and then divide it.

Look at this example.

Change 0.6 to a common fraction.

$$0.6 = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}} = \frac{3}{5}$$

(dividing by 2) (simplest form)

Exercises

Change into fractions.

a) $0.4 = \frac{4}{10} = \frac{2}{5}$

b) $0.25 = \frac{25}{100} = \frac{5}{20} = \frac{1}{4}$

c) $0.04 = \frac{4}{100} = \frac{1}{25}$

d) $0.16 = \frac{16}{100} = \frac{4}{25}$

e) $0.245 = \frac{245}{1000} = \frac{49}{200}$

f) $0.0006 = \frac{6}{10000} = \frac{3}{5000}$

4.4 Multiplying Decimals by Whole Numbers

Multiplying decimal fractions by whole numbers is easy. We have to remember to put the decimal point in its correct place.

Look at this example.

Multiply 4.7 by 9.

```
  4.7   ←  1 decimal place
x   9     (↑ align)
-----
 42.3   ←  1 decimal place
```

Multiply in the same way as we do for whole numbers.

To put the decimal point in the answer, count the number of decimal places as they are in the decimal fraction.

Exercises

Multiply the following.

a) 3.9 × 4 = 15.6

b) 4.4 × 8 = 35.2

c) 9.62 × 6 = 57.72

d) 3.02 × 7 = 21.14

e) 3.201 × 2 = 6.402

f) 65.286 × 7 = 457.002

4.5 Dividing Decimals by Whole Numbers

When we divide a decimal fraction by a whole number, we divide it the same way as whole numbers. We must remember to put the decimal point in the answer directly above the decimal point of the decimal fraction (dividend).

Example

Divide 15.6 by 3.

```
   5.2
3 | 15.6
  - 15
    .6
  -  .6
    0
```

Divide the same way as whole numbers.

Place the decimal point in the answer directly above the decimal point in the dividend.

Exercise

Solve the following

a) 4 | 8.92 = 2.23

```
 8.92 | 4
-8    2.23
 0.9
-0.8
 1.2
-1.2
 0
```

b) 4 | 56.8 = 14.2

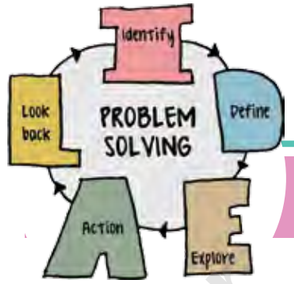
```
 56.8 | 4
-56   14.2
 0.8
-0.8
 0
```

c) 3 | 2.79 = 0.93

```
 2.79 | 3
-2.7   0.93
 0.9
-0.9
 0
```

d) 9 | 76.86 = 8.54

```
 76.86 | 9
-72     8.54
 4.8
-4.5
 0.36
-0.36
 0
```

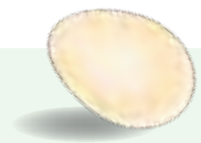
4.6 Problem Solving

Remember

1- Read
2- Plan
3- Solve
4- Check

1- Sam needs $3.40. He has $0.75. How much more money does he need? $3.40 - $0.75 = $2.65

2- George walks 0.8 km to school and Nina walks 0.4 km to school. How much farther than Nina does George walk? 0.8 km - 0.4 km = 0.4 km



3- One egg weighs 50.22 grams. How much do 5 pencils weigh?

50.22 * 5 = 251.1

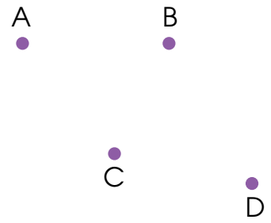
4- Lana used 1.75 cup of sugar in a batch of brownies. What fraction of a cup did she use? $\frac{7}{4}$

5- A driving class begins at 8:15 a.m. It lasts for 5 hours and 30 minutes. At what time will the class end? 1 : 45 pm

5.1 Lines

A point: is an exact position or location on a plane surface.

Does it have a site? Does it have a position?



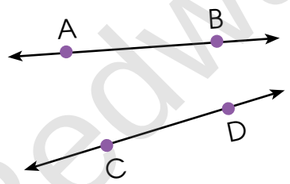
All of these are points

Line: A long thin mark made by a pen, pencil, etc. A line:

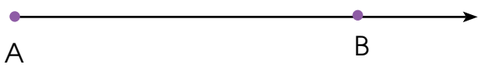
- is straight (no curves).
- has no thickness.
- extends in both directions without an end.

Line 1 = $\overleftarrow{AB}$

Line 2 = $\overleftrightarrow{CD}$



Ray: Look at the line below:



It starts from point A and goes to meet point B, but it does not stop as it has no end point. The arrowhead tells us that the line is going on. It is called a '**ray**' and is written as $\overrightarrow{AB}$. It can also be written as $\overleftarrow{BA}$. A ray is a part of a line. It has only one end point.

Line Segment:

It is a part of a line that connects two points; it has a definite end point.

The word "segment" indicates that the line has ends.

Example:

Called $\overline{AB}$

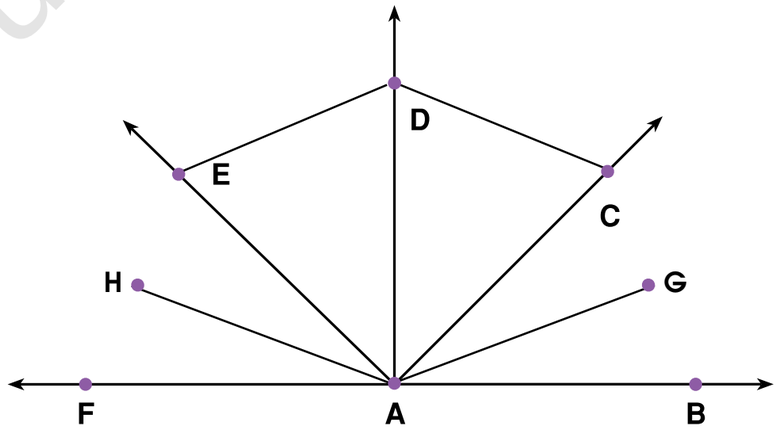
Exercises

Look at this figure and write names of:

1- Lines FB

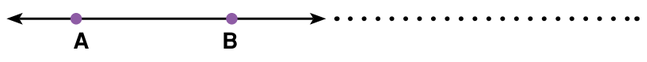
2- Rays AF, AC, AB, AD, AE

3- Line segments AG, AH

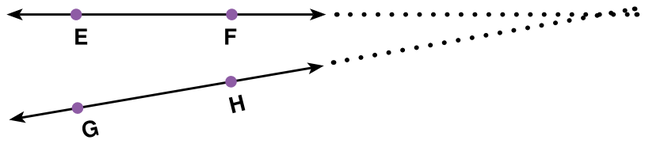


5.2 Parallel Lines

(1)



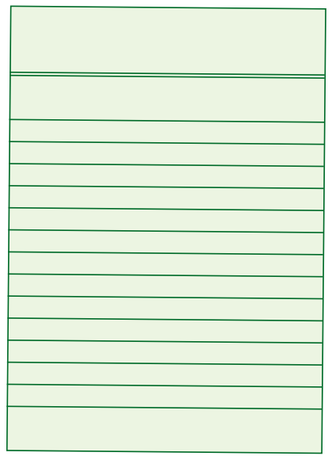
(2)



Join the dots to extend the lines in figures (1) and (2).
Did the lines in figure (1) meet each other? What is the distance between AC, BD?
Did the lines in figure (2) meet each other? What is the distance between EG, FH?

Two lines that lie in the same plane that never meet or cross each other are called `parallel lines'

So, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines.



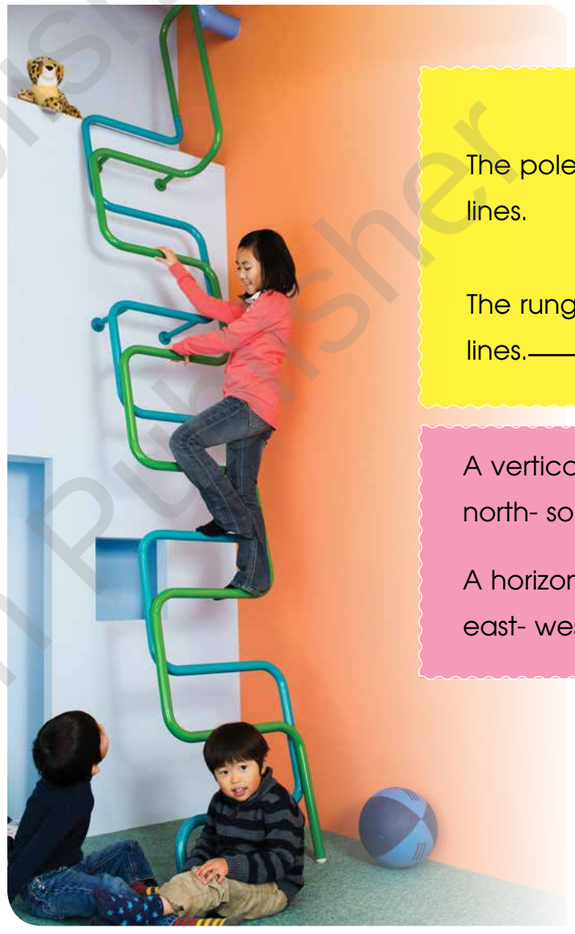
Look at the lines on the page.
All the lines on this page are parallel lines.

Exercise

Give examples from your classroom, home and your life of parallel lines and share them with your classroom

5.3 Vertical and Horizontal Lines

Look at the ladder standing against the wall.



The poles of the ladder are vertical lines.

The rungs of the ladder are horizontal lines.

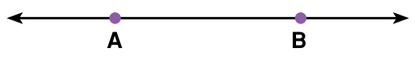
A vertical line always lies on the (up - down) north- south axis.

A horizontal line always lies on the (left- right) east- west axis.

Exercises

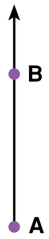
1 - Name the following lines. Classify them as either (vertical, horizontal, different).

a)



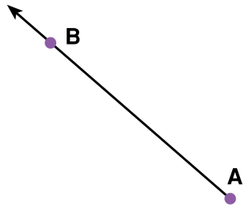
Horizontal

b)



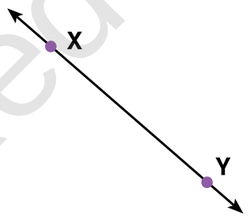
Vertical

c)



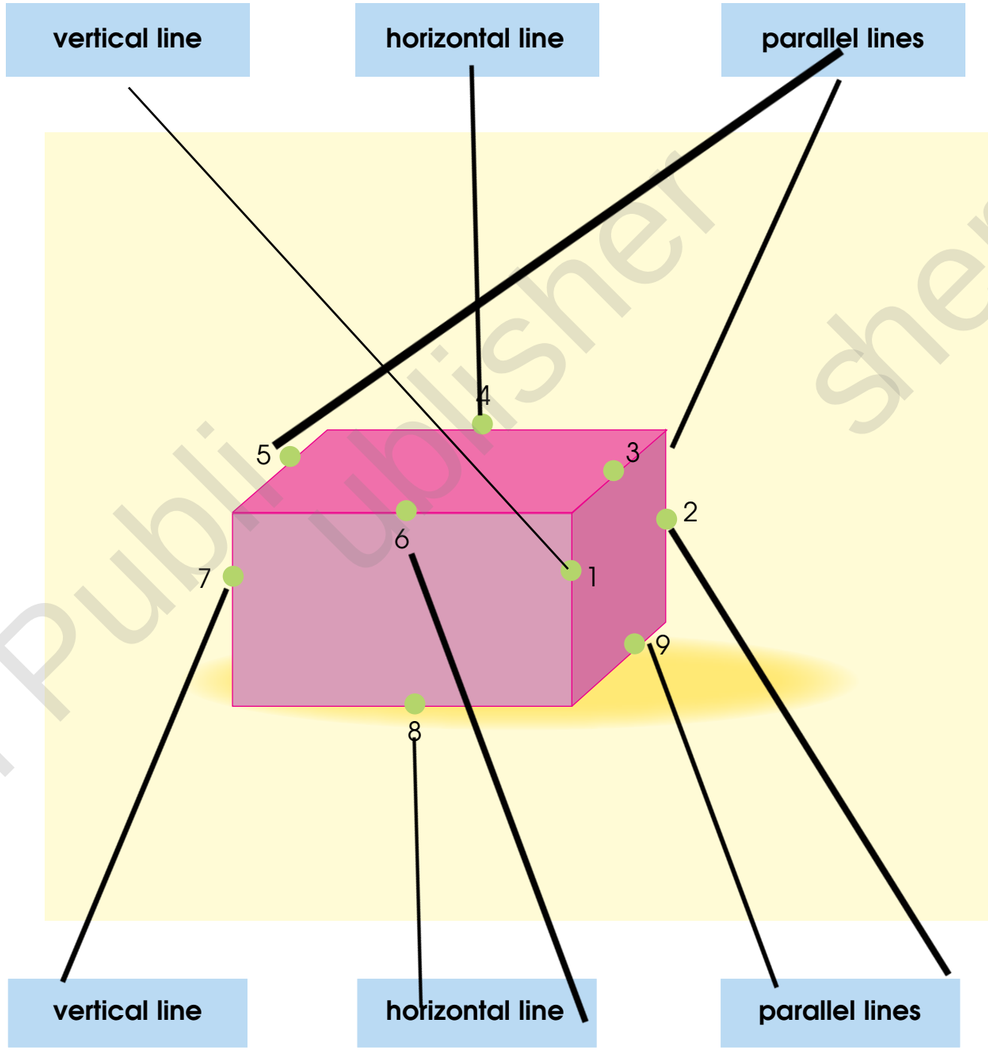
different

d)



different

2 - Look at the shape; imagine that it is your classroom now. Write or match the numbered lines with their names.

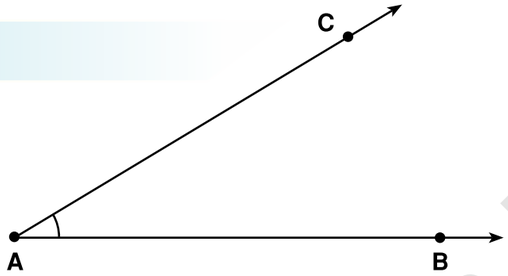


5.4 Angles

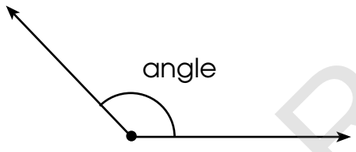
- What is an angle?
- What are the components of an angle?
- Are there types for angles?
- How can you measure an angle?
- What is the unit of measurement used for angles?

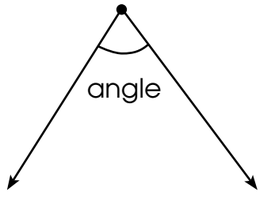
Angle

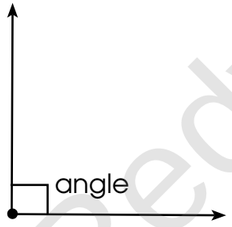
Look at the figure on the right.



We can see two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ starting from a common end point A.
Two rays with a common end point form an angle.
So, can you define the angle?







These are all different forms of angles.

A protractor is an angle measurement tool. It's like a ruler, and it is used to measure the length of lines.

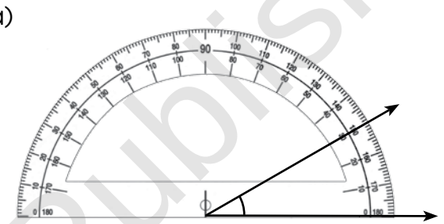
The unit that is used in a ruler is centimeter (cm) or millimeter (mm), or we may use meter (m).

But with **angles**, we use a unit called **degree**. (The protractor gives us a degree) ($^{\circ}$).

Exercise

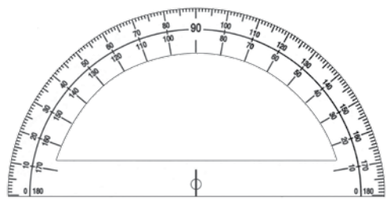
Look at the angles below and write the measurement of each.
(Don't forget the unit).

a)



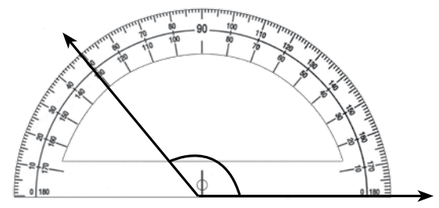
30°

b)



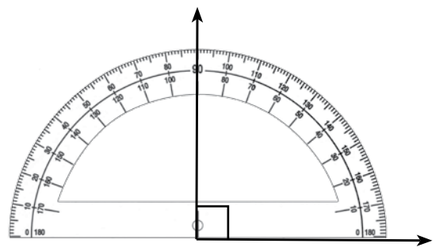
0°

c)



130 °

d)

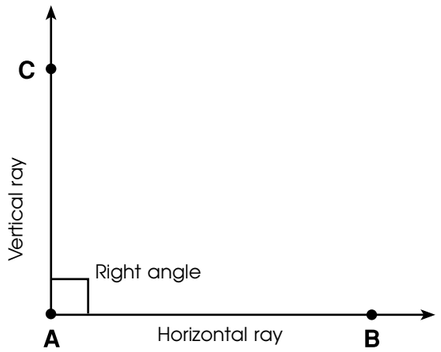


90°

5.5 Types of Angles

1- Right Angle

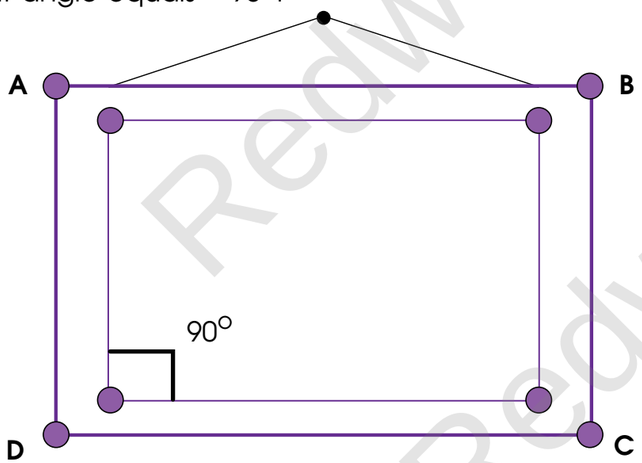
When a horizontal ray and a vertical ray start from a common end point, a right angle is formed.



We use the ⌝ sign only with right angles.

See the picture frame below.

The measurement of a right angle equals = 90^{o}.



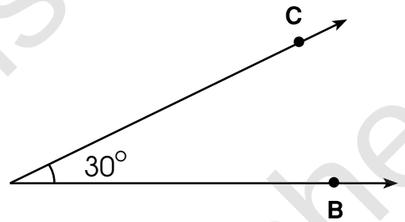
The corners of the frame (A, B, C, D) are right angles.

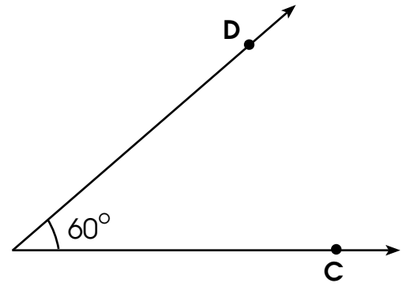
check it by a protractor.

Give us examples from your real life on right angles.

2- Acute Angle

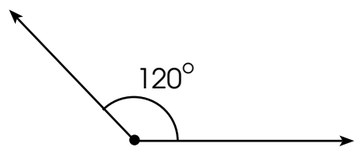
The angle that has a measurement less than 90^{o} is a sharp angle, such as 10^{o}, 30^{o}, 45^{o}, 60^{o}.

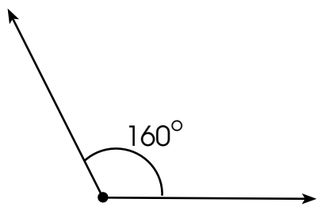




3- Obtuse Angle

An angle that has a measurement between 90^{o} and 180^{o}, such as, 100^{o}, 120^{o}, 160^{o}.

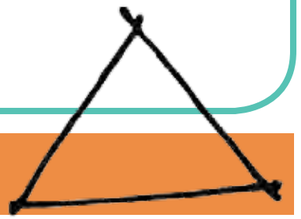




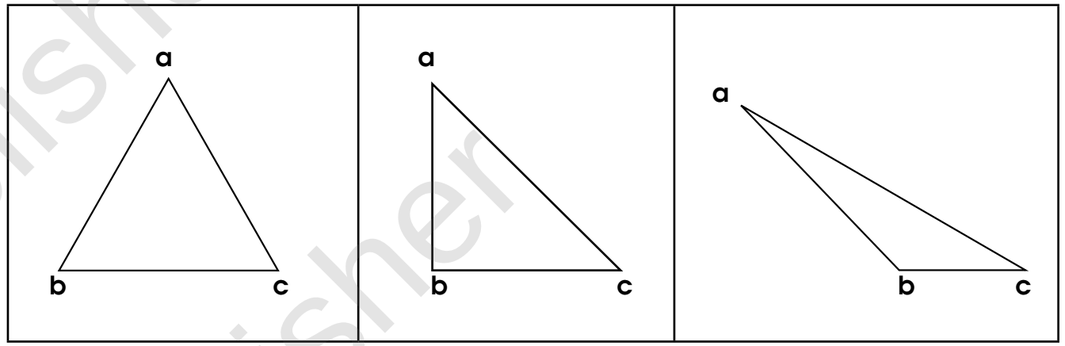
Exercise

Use your ruler and pencil to draw acute, right and obtuse angles. Then, find the measurement of each angle.

5.6 Triangles



You already know that a triangle is a closed shape which has three sides and three angles. You can see that there are 3 angles in a triangle.

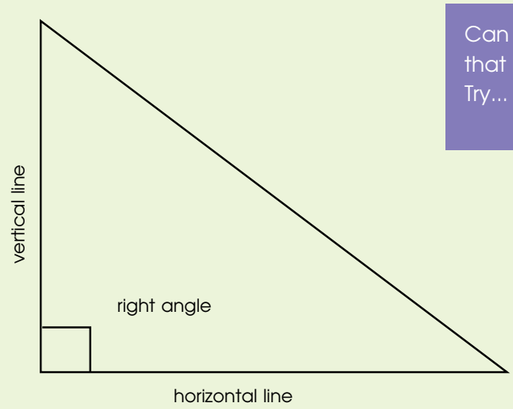


Find the measurement of every angle in each triangle shown above.
What do you notice?

1- Right Angled Triangle

If **one angle** of a triangle is a right angle made up of vertical and horizontal lines, then the triangle is called a 'right-angled triangle'.

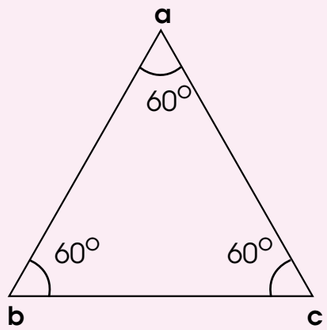
Can you draw a triangle that has two right angles? Try...



2- Acute Angles Triangle

Notice that here we are talking about all the angles of a triangle and not just a single one like the right angled triangle.

So when we draw an acute angles triangle all angles will be acute angles.



∢ a = 60°

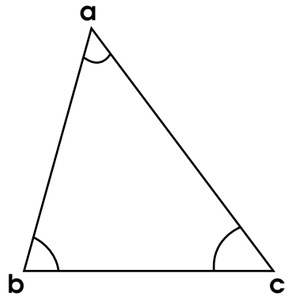
∢ b = 60°

∢ c = 60°

All are acute angles

Exercises

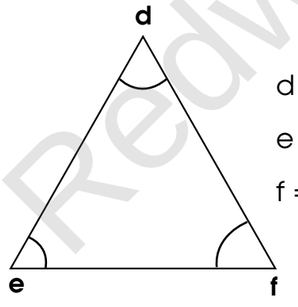
1- Find the measurement of the missing angle in these triangles.



a = 30°

b = 70°

c = 80°



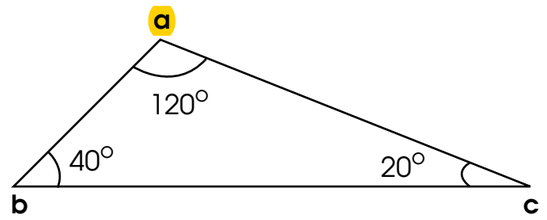
d = 65°

e = 57°

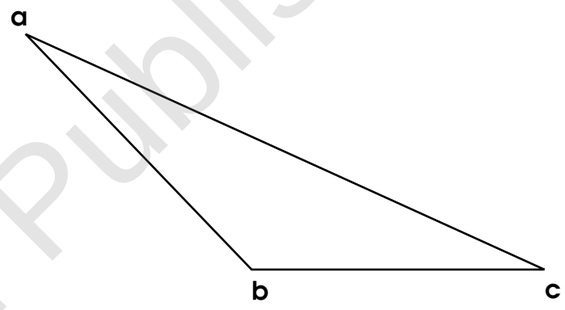
f = 58°

2- Draw two acute angles triangles; show the measurements.

3 - The obtuse triangle here has exactly one obtuse angle (Obtuse angle: the angle whose measurement is greater than 90°).



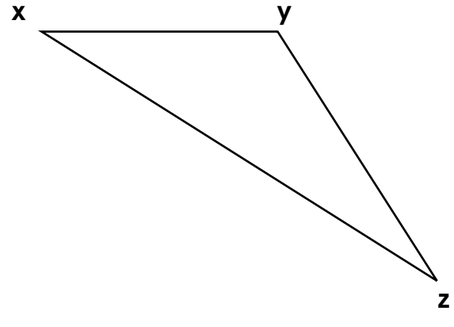
4- Can you estimate a logical measurement for each angle in these triangles?



a = 30°

b = 120°

c = 30°



x = 40°

y = 110°

z = 30°

6.1 Metric System (Units of Length)

The most common metric units of length are the kilometer (km), the meter (m), the centimeter (cm) and the millimeter (mm); units of length are related as follows:

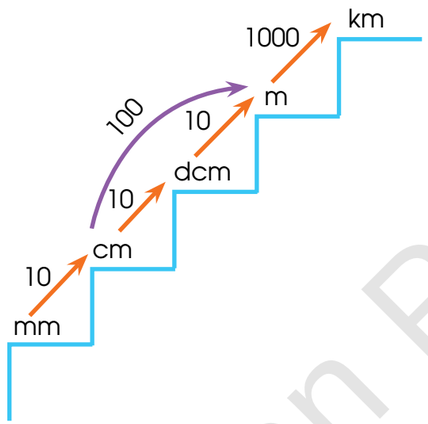
1 centimeter =10 millimeters

1 meter =100 centimeters

1 meter =1000 millimeters

1 kilometer =1000 meters

1 decimeter =10 centimeters



How big are metric units of distance?

- A head of a pin is 1 millimeter thick.
- A finger nail is about 1 centimeter wide.
- The length of a guitar is about 1 meter.
- A kilometer is equal to 1000 meters. It is a little over half a mile.

Use the suitable unit to measure the following.

1- Width of a road? m

2- Distance between two cities? km

3- Length of this paper? cm

4- Width of a pin? mm

5- Length of your mobile? cm

6- Length of your tablet? cm

7- Length of the airport road? km

- Every 1(m) = 100 (cm), but 5 (m) = 500 (cm).
- Every 1(km) = 1000 (m), but 7(km) = 7000 (m).
- Every 1(cm) =10 (mm), but 3 (cm) = 30 (mm).
- Every 1(m) = 10 (dcm), but 10 (m) = 100 (dcm).

Exercises

1- Convert the following measurements to the units indicated.

a) 6 m to mm 6 * 1000 = 6000

b) 4 km to cm 4000 * 100 = 40000

c) 2 km to mm 2000 * 1000 = 2000000

d) 7.2 m to cm 7.2 * 100 = 720

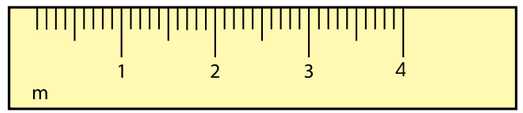
e) 9.23 m to mm 9.23 * 1000 = 9230

f) 58.6 m to cm 58.6 * 100 = 5860

g) 4.007 km to m 4.007 * 1000 = 4007

h) 0.0315 cm to mm 0.0315 * 10 = 0.315

2- On the other side, look at the ruler.



Do you know how to represent that 1 (cm) equals 0.1 (m)?
Then 1(mm) = 0.1 (cm).
and 1(cm) = 0.01 (m).
1 (m) = 0.001 (km).

3- Find.

a) 2(cm) = 0.02 (m).

b) 4 (m) = 0.004 (km).

c) 500 (mm) = 0.5 (m).

d) 200 (cm) = 0.002 (km).

6.2 Metric System (Units of Mass)

Mass is used to measure the weight of an object, so when you want to measure the mass of your body you step on the scale.

As you see at the picture on the right, how much is shown? The answer is 40. What is the unit? The unit is kilogram.

So we say his/her weight/mass is 40 kilograms.

The most popular units that measure mass are kilogram and gram.

So (1) kilogram (kg) = 1000 grams (g).

(1) ton (t) =1000 (kg).

Gram: is a very light unit that is used with something very small that needs to be accurate. That is why you often see things measured in hundreds of grams.

Kilograms: are great for measuring things that can be lifted by people.

Ton: is good to measure things that are very heavy like cars and trucks.

Exercises

1) The mass of a laptop is 2.5 kg how many grams is this? 2.5 * 1000 = 2500 g

2) The mass of a chocolate bars is 250 g, what is the total mass of the chocolate bars in (kg)? 250 / 1000 = 0.25 kg

3) The mass of a truck is 4500 kg. (10) of those trucks are packed onto a ferry. What is the total mass of the trucks in tonnes? 4500 * 10 = 45000 kg
4500 / 1000 = 45 tonnes

4) The mass of a pencil is about 10 (g) or we can write it as 0.01 (kg).

5) The mass of an object is 15 kg.
How many grams is this? 15 * 1000 = 15000 g
How many tonnes is this? 15 / 1000 = 0.015 tonnes

6.3 Metric System Units of Volume

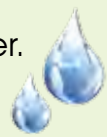
Volume measures capacity; the volume of a cup is the amount of space inside the cup or how much water it takes to fill the cup.

In the metric system, the most common units of volume are liters and milliliters. The question is how big is a liter (L)?

This bottle contains about 1 liter of liquid.

And how big is a milliliter (Ml)?

A single rain drop contains about 1 milliliter of water.



Now 1 liter = 1000 milliliters

$1 \text{ ml} = \frac{1}{1000}$

Exercises

Complete.

a) 34 L = 34000 (ml). 34 * 1000 = 34000

b) 7.5 L = 7500 (ml). 7.5 * 1000 = 7500

c) 1350 (ml) = 1.35 (L). 1350 / 1000 = 1.35

d) 40 L = 40000 (ml). 40000 / 1000 = 40

e) 42000 (ml) = 42 (L). 42000 / 1000 = 42

f) 4 (L) and 500 (ml) = 4.5 (L). 500 / 1000 = 0.5, 4 + 0.5 = 4.5